

RADAR SYSTEMS

LECTURE NOTES

B.TECH

(IV YEAR – II SEM)

(2020-21)

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MALLA REDDY COLLEGE
OF ENGINEERING & TECHNOLOGY
(Autonomous Institution – UGC, Govt. of India)

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MALLA REDDY COLLEGE OF ENGINEERING AND TECHNOLOGY**IV Year B.Tech. ECE-II Sem**

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CORE ELECTIVE – V
(R17A0429) RADAR SYSTEMS

COURSE OBJECTIVES

1. To learn Radar Fundamentals like Radar Equation, Operating frequencies & Applications.
2. To understand the basic concepts of different types of Radars for surveillance & Tracking.
3. To know the various types of tracking techniques involved.
4. To understand Radar Receivers, MTI filters, displays and antennas.

UNIT I

Basics of Radar: Introduction, Maximum Unambiguous Range, Radar Waveforms, Simple form of Radar Equation, Radar Block Diagram and Operation, Radar Frequencies and Applications, Prediction of Range Performance, Minimum Detectable Signal, Receiver Noise, Modified Radar Range Equation, Related Problems.

Radar Equation: SNR, Envelope Detector-False Alarm Time and Probability, Integration of Radar Pulses, Radar Cross Section of Targets (simple targets - sphere, cone-sphere), Transmitter Power, PRF and Range Ambiguities, System Losses (qualitative treatment), Related Problems.

UNIT-II

CW and Frequency Modulated Radar: Doppler Effect, CW Radar – Block Diagram, Isolation between Transmitter and Receiver, Non-zero IF Receiver, Receiver Bandwidth Requirements, Applications of CW radar, Related Problems.

FM-CW Radar: FM-CW Radar, Range and Doppler Measurement, Block Diagram and Characteristics (Approaching/ Receding Targets), FM-CW altimeter, Multiple Frequency CW Radar.

UNIT-III

MTI and Pulse Doppler Radar: Introduction, Principle, MTI Radar with - Power Amplifier Transmitter and Power Oscillator Transmitter, Delay Line Cancellers – Filter Characteristics, Blind Speeds, Double Cancellation, Staggered PRFs. Range Gated Doppler Filters. MTI Radar Parameters, Limitations to MTI Performance, MTI versus Pulse Doppler Radar.

Tracking Radar: Tracking with Radar, Sequential Lobing, Conical Scan, Monopulse Tracking Radar – Amplitude Comparison Monopulse (one- and two- coordinates), Angular Accuracy, Tracking in Range, Acquisition and Scanning Patterns, Comparison of Trackers.

UNIT-IV

Detection of Radar Signals in Noise: Introduction, Matched Filter Receiver – Response Characteristics and Derivation, Correlation Function and Cross-correlation Receiver, Efficiency of Non-matched Filters, Matched Filter with Non-white Noise.

UNIT-V

Radar Receivers: Noise Figure and Noise Temperature, Displays – types, Introduction to Phased Array Antennas – Basic Concepts, Radiation Pattern, Beam Steering and Beam Width changes, Applications, Advantages and Limitations.

Electronic Warfare: Introduction to ESM, ECM and ECCM systems.

TEXT BOOK:

1. Introduction to Radar Systems – Merrill I. Skolnik, TMH Special Indian Edition, 2nd Edition, Tata McGraw-Hill, 2007.

REFERENCES:

1. Introduction to Radar Systems – Merrill I. Skolnik, 3rd Edition Tata McGraw-Hill, 2001.
2. Radar: Principles, Technology, Applications-Byron Edde, Pearson Education, 2004.
3. Principles of Modern Radar: Basic Principles-Mark A. Richards, James A. Scheer, William A. Holm, Yesdee, 2013.
4. 'Radar Hand Book ' Ed. By M.I Skolnik, 2nd Edition, Tata McGraw Hill.
5. 'Understanding Radar Systems' by Simon Kinsley and Shaun Quegan, Scitech Publishing, McGraw-Hill.

COURSE OUTCOMES

1. Demonstrate an understanding of the factors affecting the radar performance using Radar Range Equation.
2. Analyze the principle of FM-CW radar and apply it in FM-CW Altimeter.
3. Differentiate between a MTI Radar and a Pulse Doppler Radar based on their working principle.
4. Demonstrate an understanding of the importance of Matched Filter Receivers in Radars.
5. Familiarize with the different types of Radar Displays and their application in real time scenario

UNIT-1(PART-A)

BASICS OF RADAR

BASICS OF RADAR

- Introduction
- Maximum unambiguous Range
- Radar Block diagram and operation
- Simple form of the radar equation
- Radar frequencies and Applications
- Prediction of range performance
- Minimum Detectable signal
- Receiver Noise
- Modified Radar Range equation
- Illustrative problems

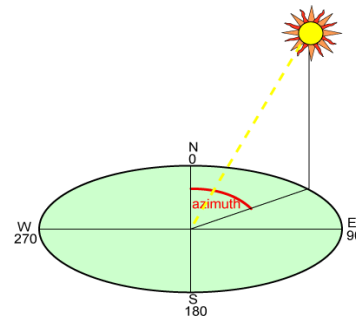
RADAR is an Acronym for Radio Detection And Ranging

Radar is a detection system that uses **radio waves** to determine the range, angle, or velocity of objects. It can be used to detect **aircraft**, **ships**, **spacecraft**, **guided missiles**, **motor vehicles**, **weather formations**, and **terrain**.

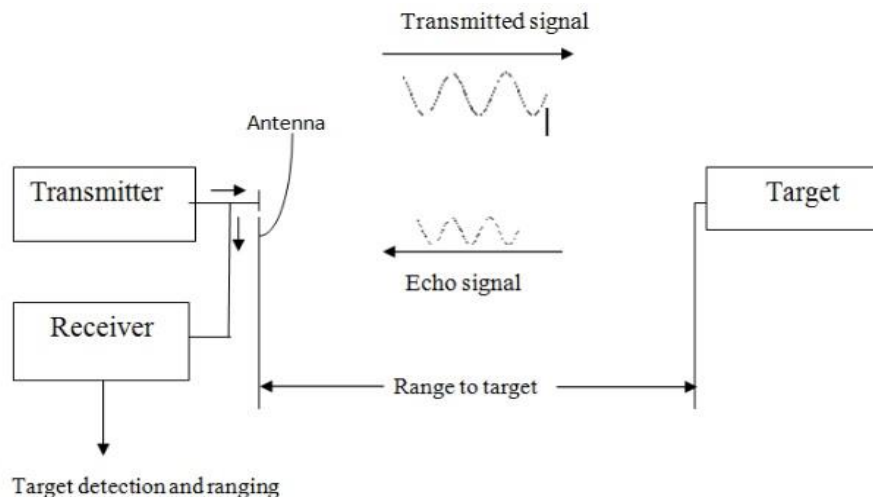
A radar system consists of a **transmitter** producing **electromagnetic waves** in the **radio** or **microwaves** domain, a transmitting **antenna**, a receiving antenna (often the same antenna is used for transmitting and receiving) and a **receiver** and **processor** to determine properties of the object(s).

FUNCTIONS OF RADAR

- Detects the presence of target
- Gives the range of the target from the Radar station
- Gives the azimuth angle and elevation angle of the target
- Gives the radial velocity of target.



PRINCIPLE OF WORKING



Two basic radar systems exist

1. Monostatic
2. Bistatic

Measurement of Range

PRF: The no of radar pulses transmitted per second is known as pulse repetition frequency or pulse repetition rate. F_p

PRT: The time from beginning of first pulse to the beginning of the next is called pulse repetition time. T_p

$$F_p = \frac{1}{T_p}$$

T_R --Time taken by EM pulse to travel to target and come back to same antenna

R- range of target

C –Velocity of EM waves = 3×10^8 Meters/sec

Rest Time or Receiver Time : The time between two successive transmitted pulse is called as Rest Time or Receiver Time

Radar range determination

- The most common radar waveform is a train of narrow, rectangular-shape pulses modulating a sine wave carrier.
- The distance, or range, to the target is determined by measuring the time T_R taken by the pulse to travel to the target and return.
- Electromagnetic energy in free space travels with the speed of light c (3×10^8 m/s) therefore range **R** is given by : **$R = cT_R / 2$**
- The range **R** in kilometers or nautical miles, and T_R in microseconds, the above relation becomes: **$R(\text{km}) = 0.15 \times T_R (\mu\text{S})$ or $R(\text{nmi}) = 0.081 \times T_R (\mu\text{S})$**
- Each microsecond of round-trip travel time corresponds to a distance of **0.081** nautical mile, **0.093** statute mile, **150** meters, **164** yards, or **492** feet.
- (1 mile = 0.8689 nautical mile or 1.6 km
1 nautical mile = 1.15078 miles or 1.8412 km)
- It takes 12.35 μs for radar signal to travel a nautical mile and back

Maximum unambiguous range

- Once signal is transmitted into space by a radar, sufficient time must elapse to allow all echo signals to return to the radar before the next pulse is transmitted.
- The rate which pulses may be transmitted T_p is determined by the longest range at which the target is expected.
- If the time between the pulses T_p is too short, the echo signal from target may arrive after the next pulse transmitted and it leads to incorrect or ambiguous measurement of the range.
- The echoes that arrive after the transmission of next pulse are called **second time around echoes or second return echoes**.
- The maximum range from which a transmitted radar pulse can be reflected and received before the next pulse is transmitted.

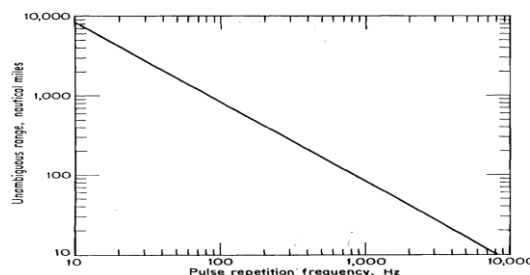
or

the range beyond which targets appear as second time around echoes is called the **maximum unambiguous range**.

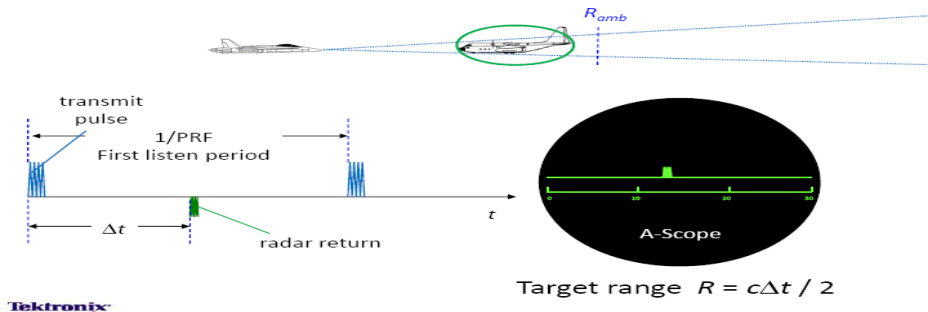
- $R_{\max}$ is the farthest target range that can be detected by a Radar without ambiguity and is also called Maximum Unambiguous Range of the Radar. Since PRF $f_p = 1/T_p$ It is also given by :

$$R_{\text{un}} \text{ or } R_{\max} = CT_p/2 = C/2f_p$$

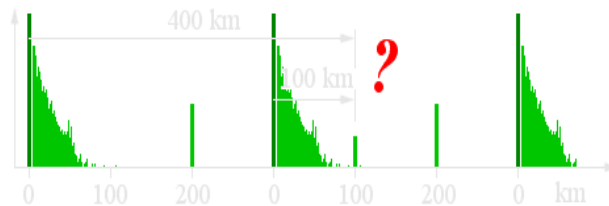
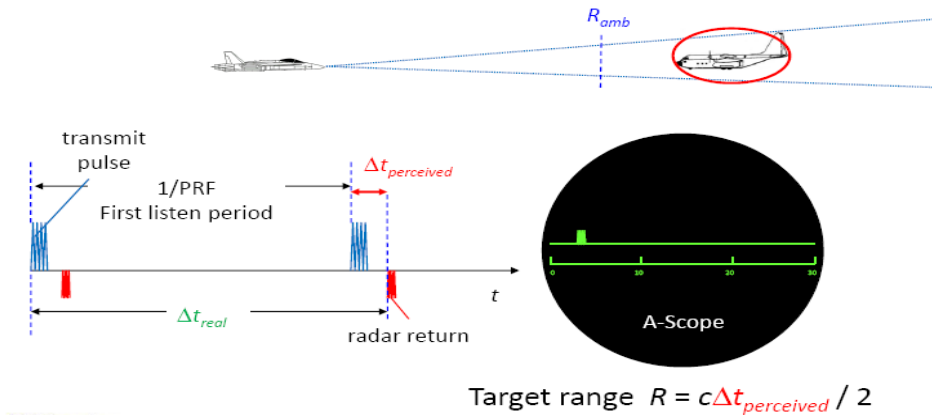
- If the range of target is more than the Maximum Unambiguous Range, multiple time around echoes occur and range computed would be erroneous.
- The relation between PRF and Maximum Unambiguous Range is linear and shown in the next slide.



Range Ambiguity – True Range



Range Ambiguity – False Range



The first transmitted pulse, after being reflected from the target in 200 km, is received by the radar before the second pulse is transmitted. There will be no ambiguity here as the reflected pulse can be easily identified as a reflection of the first pulse. But in same Figure, we notice that the reflection of a target of the first pulse is received after the second pulse has been transmitted (in range of 400 km). This causes some confusion since the radar, without any additional information, cannot determine whether the received signal is a reflection of the first pulse or of the second pulse. This leads to an ambiguity in determining the range, this received echo signal be mistaken as a short-range echo of the next cycle.

Therefore maximum unambiguous range R_{max} is the maximum range for which $t < T$.

$$R_{max} = c_0 \cdot (T - \tau) / 2 \quad \text{where } R_{max} = \text{Unambiguous Range in [m]}$$

c_0 = Speed of light [$3 \cdot 10^8$ m/s]

T = Pulse Repetition Time [sec]

τ = length of the transmitted pulse

The simple form of the radar equation

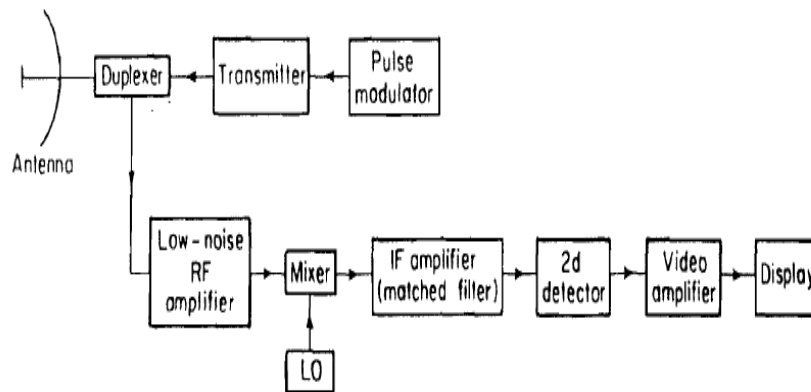
- The radar equation Relates the range of a Radar to the characteristics of the transmitter, receiver, antenna, target, and environment.
- It is useful not only for determining the maximum range, but it can serve for understanding the factors affecting radar performance.

Limitations:

- Does not adequately describe the performance of practical radar.
- Many important factors that affect range are not explicitly included.
- In practice, the observed maximum radar ranges are usually much smaller than what would be predicted by the above equations, sometimes by as much as a factor of two.

There are many reasons for the failure of the simple radar equation to correlate with actual performance and these will be explained subsequently in the modified Radar range equation .

Radar block diagram



There are two sections of radar

1. Transmitter section
2. Receiver section

Radar shown in the block diagram is called monostatic Radar since same antenna is used for transmission and reception.

Transmitter section

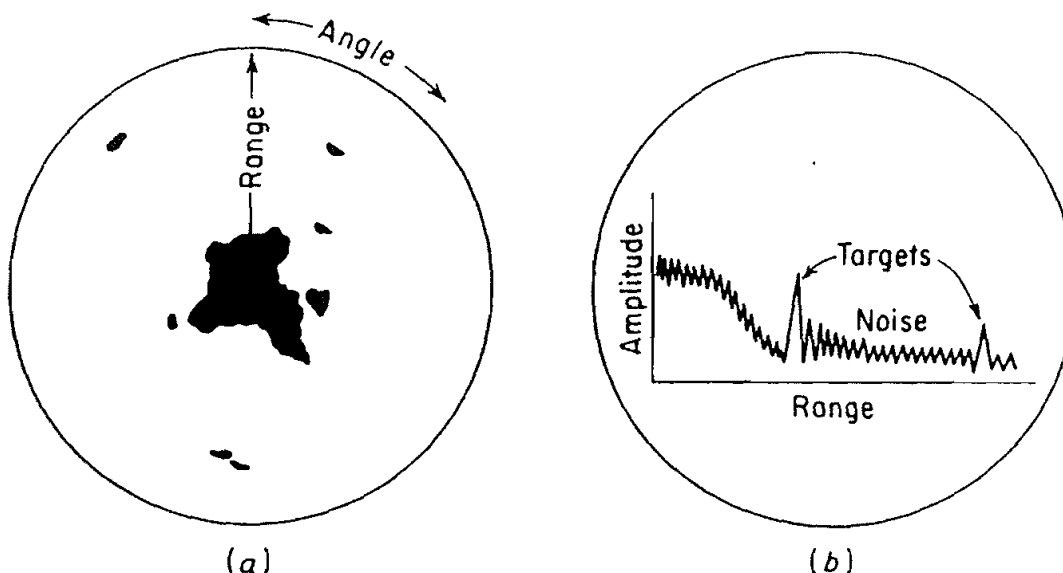
- **Transmitter** : the transmitter may be a power amplifier such as klystron, travelling wave tube or transistor amplifier. This will generate the Electrical energy at R.F.(Radio Frequency).
- **Pulse modulator** : The power amplifier (Such as Klystron, TWT) produces a high power signal, may be in terms of megawatts. Pulse modulator shown in the block is used as a switch, which will turn on and off the power amplifier.
- **Wave form generator**: A low power signal is produced by the waveform generator which is given as an input to the power amplifier.
- **Duplexer**: The duplexer allows a single antenna to be used on a time shared basis for both transmitting and receiving. The duplexer is generally a gaseous device that produces a short circuit at the input to the receiver when the transmitter is operating, so that high power flows to the antenna and not to the receiver. On the reception, the duplexer directs echo signal to the receiver and not to the transmitter. Solid state ferrite circulators and receiver protector devices can also be part of the duplexer

Receiver section:

- **Low noise RF amplifier**: The receiver is almost always a super heterodyne. LNA is used immediately after the antenna. This reduces the Noise Figures and produces the RF pulse proportional to the transmitted signal.
- **Mixer and local oscillator**: It converts the RF signal to an intermediate frequency where it is amplified by the IF amplifier. The IF frequency might be 30 or 60 MHz.
- **IF amplifier**:
 - i) It amplifies the IF pulse.
 - ii) IF amplifier is designed as a matched filter which maximizes the output peak signal to mean noise ratio.
 - iii) The matched filter maximizes the detectability of weak echo signals and attenuates unwanted signals.
 - iv) The signal bandwidth of super heterodyne receiver is determined by the bandwidth of its IF stage.
 - v) For example when pulse width is of the order of $1\mu\text{s}$ the IF bandwidth would be about 1MHz.

- **Second Detector:** the IF amplifier followed by a crystal diode which is called the second detector or demodulator. Its purpose is to assist in extracting the echo signal modulation from the carrier. It is called as 2ndDetector since it is the second diode used in the chain. The first diode is used in the mixer. Output of the 2ndDetector is the Video Pulse.
- **Video amplifier:** It is designed to provide the sufficient amplification to rise the level of the input signal to a magnitude where it can be display (CRT or Digital computer).
- **Threshold decision:** The output of video amplifier is given to the threshold detector where it is decided whether the received signal is from a target or just because of the presence of noise.
- **Display:** The Display is generally a CRT (Cathode Ray Tube)
 - (a) 'A' scope (b) PPI
 - i) 'A' scope provided Range and Echo power.
 - ii) PPI measures Range and bearing (azimuth angles)
 - iii) In addition there are other displays like 'B' scope, 'D' scope etc.

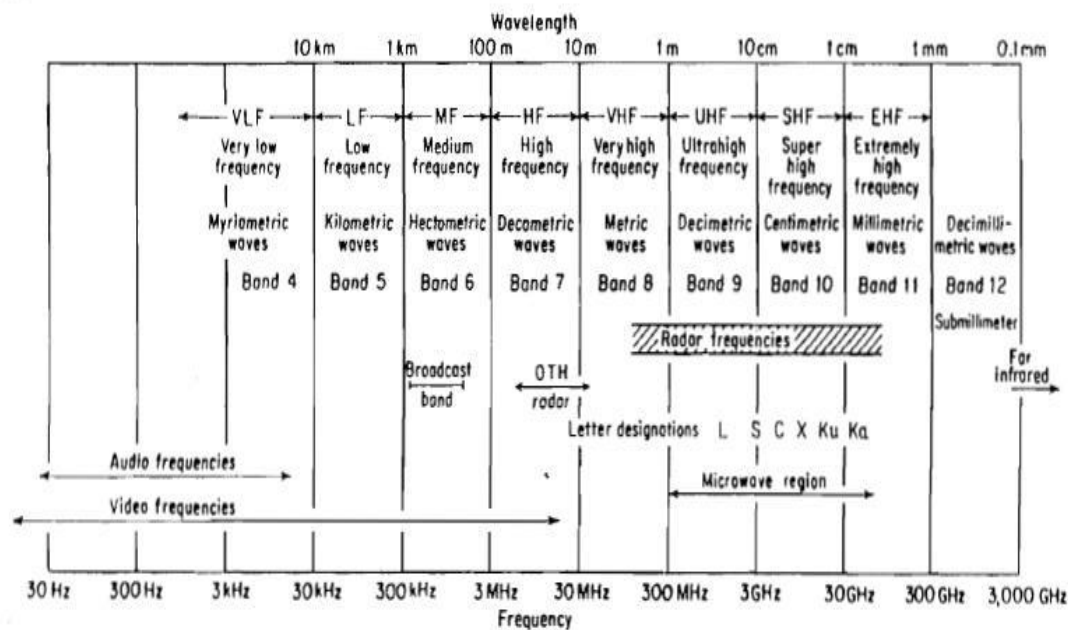
- (a) PPI presentation displaying Range vs. Angle (intensity modulation)
 (b) A-scope presentation displaying Amplitude vs. Range (deflection modulation)



RADAR FREQUENCIES

- RF spectrum is very scarce and as such Radars are allotted only a certain frequency bands for their operation by International Telecom Union ITU
- During 2nd world war, to keep the secrecy, certain code words were used. The same designations are continued even today
- Lema Band (L) 1GHZ-2GHZ, Sierra band(S) 2GHZ-4GHZ, Charlie Band (C) 4GHZ-8GHZ, Xera Band (X) 8GHZ-12GHZ
- ITU(International Telecommunication Union) allocated a portion of these bands for Radar

ELECTROMAGNETIC SPECTRUM



Standard radar-frequency letter-band nomenclature

Band designation	Nominal frequency range	Specific radiolocation (radar) bands based on ITU assignments for region 2
HF	3–30 MHz	
VHF	30–300 MHz	138–144 MHz 216–225
UHF	300–1000 MHz	420–450 MHz 890–942
L	1000–2000 MHz	1215–1400 MHz
S	2000–4000 MHz	2300–2500 MHz 2700–3700
C	4000–8000 MHz	5250–5925 MHz
X	8000–12,000 MHz	8500–10,680 MHz
K _u	12.0–18 GHz	13.4–14.0 GHz 15.7–17.7
K	18–27 GHz	24.05–24.25 GHz
K _a	27–40 GHz	33.4–36.0 GHz
mm	40–300 GHz	

FIELDS OF APPLICATION

- MILITARY
- REMOTE SENSING
- AIR TRAFFIC CONTROL
- LAW ENFORCEMENT AND HIGHWAY
- SECURITY
- AIRCRAFT SAFETY AND NAVIGATION
- SHIP SAFETY
- SPACE
- MISCELLANEOUS APPLICATIONS

MILITARY:

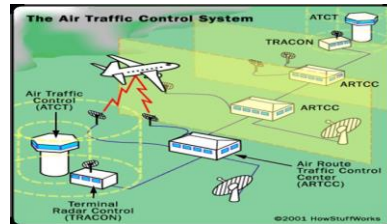
- Important part of air defence system, operation of offensive missiles & other weapons.
- Target detection, target tracking & weapon control .
- Also
- used in area, ground & air surveillance.

AIR TRAFFIC CONTROL

- Used to safely control air traffic in the vicinity of the airports and enroute.
- Ground vehicular traffic & aircraft taxing.
- Mapping of regions of rain in the vicinity of airports & weather.

LAW ENFORCEMENT & HIGHWAY SAFETY:

- Radar speed meters are used by police for enforcing speed limits.
- It is used for warning of pending collision, actuating air bag or warning of obstruction or people behind a vehicle or in the side blind zone



REMOTE SENSING

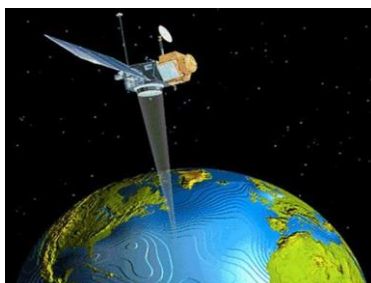
- Weather observation-t.V.Reporting
- Planetary observation
- Below ground probing
- Mapping of sea ice

AIRCRAFT SAFETY & NAVIGATION

- Low flying military aircrafts rely on terrain avoidance & terrain following radars to avoid collision with high terrain & obstructions

SHIP SAFETY

- Radar is found on ships & boats for collision avoidance & to observe navigation buoys, when the visibility is poor.
- Shore based radars are used for surveillance of harbours & river traffic.



SPACE

- Space vehicles have used radar for clocking & for landing on the moon.
- Used for planetary exploration.
- Ground based radars are used for detection & tracking of satellites & other space objects.
- Used for radio astronomy.

OTHER APPLICATIONS

- It is used for in industry for the non contact measurement of speed & distance.
- Used for oil & gas exploration.
- Used to study movements of insects & birds.



PREDICTION OF RANGE PERFORMANCE

The simple form of the radar equation derived earlier expresses the maximum radar range R_{max} in terms of radar and target parameters:

$$R_{max} = [(P_t \cdot G \cdot A_e \cdot \sigma) / (4\pi)^2 \cdot S_{min}]^{1/4}$$

where

P_t = transmitted power, watts

G = antenna gain

A_e = antenna effective aperture, m^2

σ = radar cross section, m^2

S_{min} = minimum detectable signal, watts

All the parameters are to some extent under the control of the radar designer, except for the target cross section σ .

The radar equation states that if long ranges are desired,

1. The transmitted power must be large,
2. The radiated energy must be concentrated into a narrow beam (high transmitting antenna gain),
3. The received echo energy must be collected with a large antenna aperture (also synonymous with high gain) and
4. The receiver must be sensitive to weak signals.

In practice, however, the simple radar equation does not predict the range performance of actual radars. The predicted values of radar range are usually optimistic. In some cases the actual range might be only half of that is predicted.

The failure of the simple form of radar equation is due to

1. The statistical nature of the minimum detectable signal determined by receiver noise.
2. Fluctuations and uncertainty in radar cross-section.

3. The losses throughout the radar system.
4. Propagation effects caused by the earth's surface and atmosphere.

Because of statistical nature of receiver noise and target cross section, the maximum radar range is described probabilistically rather than single number.

Therefore the radar range equation includes

1. Probability that radar will detect a target at a particular range(p_d).
2. Probability of making a false detection when no target is present(p_{fa}).

From the above facts it can be concluded that the range of radar is a function of probability of detection(p_d) and probability of false alarm(p_{fa}). The prediction of radar range is not accurate as there is uncertainty in various parameters. Still radar range equation is an important tool for i) Assessing the performance of radar ii) Generating technical requirements and Determining system tradeoffs for designing new radar systems.

MINIMUM DETECTABLE SIGNAL

- The ability of a radar receiver to detect a weak echo signal is limited by the noise present in the frequency spectrum.
- The weakest signal that the receiver can detect is called the **minimum detectable signal**. It is difficult to define what is **minimum detectable signal** (MDS) because of its statistical nature and the criterion for deciding whether a target is present or not may not be too well defined.
- Detection is normally based on establishing a threshold level at the output of the receiver (as shown by the dotted line). Whenever Rx output signal which is a mixture of echo and noise crosses this threshold then it is detected as a target. This is called **threshold detection**.
- Consider the output of a typical radar receiver as a function of time as shown in the figure below which typically represents one sweep of the video output displayed on an A-scope.

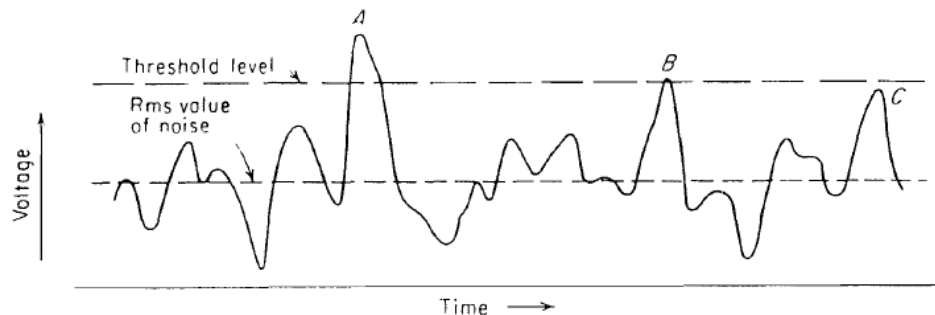


Fig : Typical envelope of a radar receiver output as a function of time. A, B, and C are three targets representing signal plus noise. A and B are valid detections, but C is a missed detection

1. If the threshold level were set properly, the signal would not generally exceed the threshold if noise alone were present, but would exceed it if a strong signal were present along with the noise.
2. If the threshold level is set too low, noise might exceed it and be mistaken for a target. This is called a false alarm.

3. If the threshold level were set too high, noise might not be large enough to cause false alarms, but weak target echoes might not exceed the threshold and would not be detected. This is called missed detection.
4. Here points A,B and C represents signal plus noise.
5. The signal at **A** is large which has a much larger amplitude than the noise. Hence target detection is possible without any difficulty and ambiguity.
6. Next consider the two signals at **B** and **C**, representing target echoes of equal amplitude. The noise voltage accompanying the signal at **B** is large enough so that the combination of signal plus noise exceeds the threshold and target detection is still possible. Thus the presence of noise will sometimes enhance the detection of weak signals.
7. But ,for the target **C** , the noise is not as large and the resultant signal plus noise does not cross the threshold and hence target is not detected.

- **Threshold Level setting:** Weak signals such as C would not be lost if the threshold level were lower. But too low threshold causes false alarms. If the threshold is set too low, false target indications are obtained, but if it is set too high, targets might be missed. The selection of the proper threshold level is necessary to avoid the mistakes of

- 1.Failing to recognize a signal that is present (missed detection) or
- 2.Falsely indicating the presence of a signal when it does not exist (false alarm)

The signal-to-noise ratio is a better measure of a radar's detection performance than the minimum detectable signal.

RECEIVER NOISE

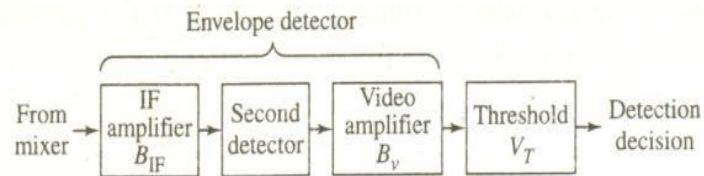
- Noise is an unwanted EM energy which interferes with the ability of the receiver to detect the wanted signal thus limiting the receiver sensitivity.
- It may originate within the receiver itself or it may enter via the receiving antenna along with the desired signal.
- If the radar were to operate in a perfectly noise free environment so that no external noise accompany the target signal.
- If the receiver itself were so perfect that it didn't generate any excess noise, there would be still be noise generated by the thermal motion of the conduction electrons in the ohmic portion of the receiver i/p stages. This is called **Thermal noise or jhonson noise**.

RADAR EQUATION

- SNR
- Envelop Detector
- False Alarm time and Probability
- Integration of Radar Pulses
- Radar Cross Section of Targets (simple targets: sphere and cone sphere)
- Transmitter Power
- PRF and Range Ambiguities
- System Losses (qualitative treatment)

SNR

- Signal to noise ratio is very important as far as radar is concerned. Because presence of target or not have small difference.
- Statistical noise theory will be applied to obtain S/N at the o/p of the IF amplifier necessary to achieve a specified prob of detection and prob of false alarm.
- **Envelope Detector:**



- Consider an IF amplifier with bandwidth B_{IF} followed by a second detector and a video amplifier with bandwidth B_v
- The second detector and video amplifier are assumed to form an envelope detector, that is one which rejects the carrier freq but passes the modulation envelop.
- To extract the modulation envelope, the video bandwidth must be wide enough to pass the low freq components generated by the second detector but no so wide as to pass the high frequency components at or near the IF.
- The video bandwidth B_v must be greater than $B_{IF}/2$ in order to pass all video modulation.

Radar Cross Section of Targets

- A radar cross section is defined as the ratio of its effective isotropic scattered power to the incident power density.

$$\sigma = \frac{\text{Power reflected towards source/Unit Angle}}{\text{Incident Power density}/4\pi}$$

$$= \lim_{R \rightarrow \infty} 4\pi R^2 \left| \frac{E_r}{E_i} \right|^2$$

where R = distance between radar and target
 E_r = strength of reflected field at radar
 E_i = strength of incident field at target

- The radar cross section depends on the characteristic dimensions of the object compared to the radar wavelength.

RCS of Simple Targets:

Sphere: A perfectly conducting sphere acts as an isotropic radiator i.e. Incident radiation scattered in all directions.

The radar cross section of the sphere is characterized into three regions

1. Rayleigh region $\frac{2\pi a}{\lambda} \ll 1$: When the wavelength is large compared to the object's dimensions is said to be Rayleigh region.
2. Optical region $\frac{2\pi a}{\lambda} \gg 1$: When the wavelength is small compared to the object's dimensions is said to be Optical region.
3. Resonance region $\frac{2\pi a}{\lambda} \approx 1$: In between the Rayleigh and Optical regions is the Resonance region where the radar wavelength is comparable to the object's dimensions.

For many objects the radar cross section is larger in the resonance region than in the other two regions.

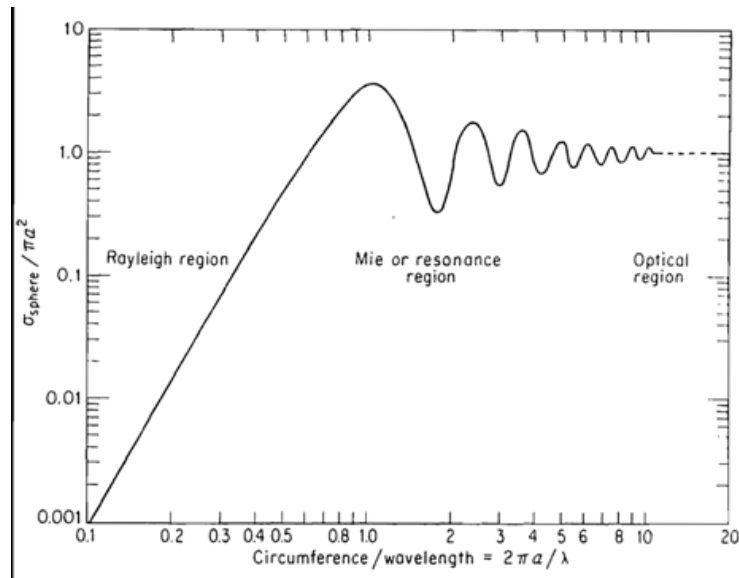
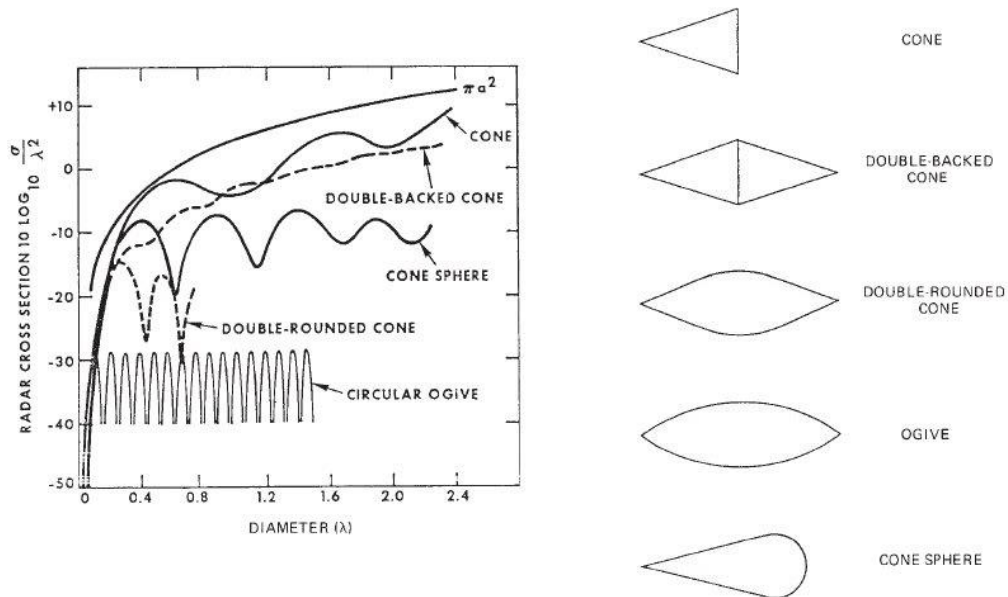


Fig: Radar cross section of a sphere as a function of circumference ($2\pi a$) measured in wavelength

- Cone sphere: It is a cone whose base is capped with a sphere. A large cross section occurs when a radar views the cone perpendicular to its surface.

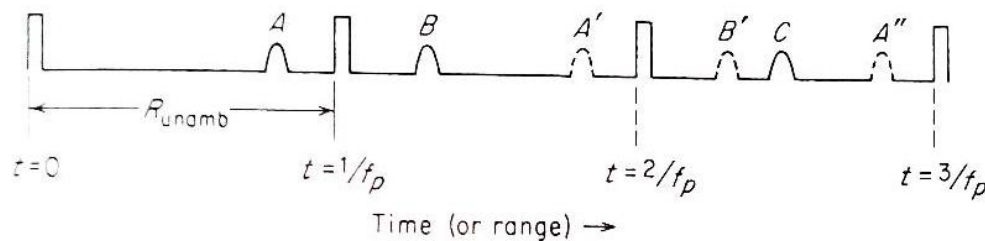


PRF and Range Ambiguities

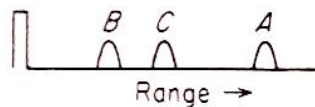
- The pulse repetition frequency (prf) is determined primarily by the maximum range at which targets are expected.

$$f_p = \frac{c}{2R_{un}}$$

- Echo signals that arrive at a time later than the pulse repetition period are known as second time around echoes or multiple time around echoes. These echoes may cause error and confusion. Also it can mask unambiguous target echoes at shorter ranges.
- Pulse Doppler radars have usually problem of range ambiguities because of prf.
- Consider the three targets located at three different positions A,B and C
- Target A is located within the maximum unambiguous range $R_{unamb} [= C.TP/2]$ of the radar, target B is at a distance greater than R_{unamb} but less than $2R_{unamb}$ and the target C is greater than $2R_{unamb}$ but less than $3R_{unamb}$



- The appearance of the three targets on an A-scope is shown below.



The ambiguous echoes B and C look very similar to unambiguous range echo A. Out of these three echoes only the range of A is correct, for B and C are not correct.

- The ambiguous range echoes are recognized by changing the prf of the radar. When the prf is changed the unambiguous echo remains at its true range. Ambiguous range echoes appear at different apparent ranges for each prf shown in below fig



- Let if prf f_{r1} has unambiguous range $R_{unambiguous1}$ and the range corresponds to it is R_1 then the true range is given by

$$R_{true} = R_1$$

$$\text{or } R_{true} = R_1 + R_{unambiguous1}$$

$$\text{or } R_{true} = R_1 + 2R_{unambiguous2}$$

- Let if prf f_{r2} has unambiguous range $R_{unambiguous2}$ and the range corresponds to it is R_2 then the true range is given by

$$R_{true} = R_2$$

$$\text{or } R_{true} = R_2 + R_{unambiguous2}$$

$$\text{or } R_{true} = R_2 + 2R_{unambiguous2}$$

The correct range is same for two prfs. Thus two or more prfs can be used to correct range ambiguity with increased accuracy and avoiding false values.

System Losses

The losses within the radar system is called system losses. The losses in a radar system reduce the signal-to-noise ratio at the receiver output.

- Microwave plumbing losses** : There is always loss in the transmission line that connects the antenna to the transmitter and receiver. In addition there can be loss in the various microwave components, such as ***duplexer, receiver protector, rotary joints, directional couplers, transmission line connectors, bends in the transmission lines and the mismatch at the antenna.***

a) **Transmission line losses**: Generally same transmission line used for both transmission and reception , the loss to be inserted in the radar eq is twice the one way loss. At lower radar frequencies, the transmission line introduces little loss. At higher radar frequencies attenuation may not be small and may have to be taken in account. In practical the transmitter and receiver should be placed close to the antenna to keep the transmission line loss small.

b) **Duplexer loss**: the loss due to a gas duplexer that protects the receiver from the high power of the transmitter is generally different on transmission and reception. It also depends on the type of duplexer used.

- In an S-band (3000 MHz) radar, for example, the plumbing losses might be as follows:

100 ft of RG-113/U A1 waveguide transmission line (two-way)	: 1.0 dB
Duplexer loss	: 2.0 dB
Loss due to poor connections (estimate)	: 0.3 dB
Rotary-joint loss	: 0.8 dB
Other RF devices	: 0.4 dB
Total plumbing loss	: 4.5 dB

2. Antenna losses:

a) Beam shape loss: In radar equation antenna gain is assumed as constant at its maximum value but in practice as a search antenna scans across a target, it does not offer its peak gain to all echo pulses. When the system integrates several echo pulses maximum antenna gain occurs when the peak of antenna beam is in direction of target.

- The beam shape loss is computed by,

$$\text{Beam shape loss} = \frac{n}{1 + 2 \sum_{k=1}^{\left(\frac{n-1}{2}\right)} e^{-\left[\frac{5.55k^2}{(n_B-1)^2}\right]}} \quad \dots (2.8.1)$$

Where,

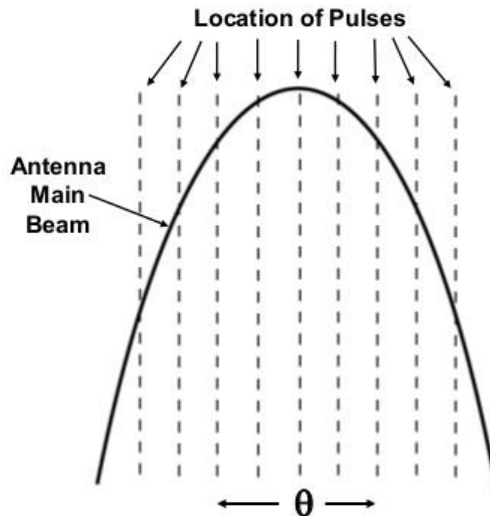
n is total number of pulses integrated.

n_B is number of pulses received within one-way half power beamwidth (θ_B).

θ_B is half power beamwidth.



Nature of Beam Shape Loss



Radar Equation assumes n pulses are integrated, all with gain G .

Except for the pulse at the center of the beam, the actual pulses illuminate the target with a gain less than the maximum.

(Adapted from Skolnik, Reference 1, p 82)

b) Scanning loss:

When a radar antenna scans rapidly compared to round trip time of the echo signal, the antenna gain may not be same for transmission and while receiving of echoes. This results in the direction of additional loss called the Scanning loss.

The scanning loss is most significant in long range scanning radars, such as space surveillance and ballistic missile defense radars.

c) Radome:

The loss introduced by radome is decided by its type and operating frequency. A commonly used ground based metal space frame radome offers a loss of 1.2dB for two way transmission.

d) Phased array losses:

Some phased array radars have additional transmission line losses due to the distribution network that connects the receiver and transmitter to multiple elements of array. These losses reduces antenna power gain.

3. Signal Processing Losses:

For detecting targets in clutters and in extraction information from the radar echo signals is very important and lossless signal processing is necessary. Various losses accounted during signal processing are

No.	Process / Components	Loss
1.	Non matched filter	0.5 to 1.0 dB
2.	Constant False-Alarm Rate (CFAR)	> 2.0 dB
3.	Automatic integrator	1.5 to 2.0 dB
4.	Limiting loss	1 dB
5.	Straddling loss	1.0 to 2.0 dB
6.	Sampling loss	2.0 dB
7.	Threshold level loss	Upto 1 dB

2.8.4 Collapsing Loss

- When additional noise samples are integrated with signal + noise pulses, this added noise causes degradation called *collapsing loss*.
- The collapsing loss is given by L_c

$$L_{c(m,n)} = \frac{L_{i(m+n)}}{L_{i(n)}}$$

where, $L_{i(m+n)}$ is integration loss for $m + n$ pulses.

$L_{i(n)}$ is integration loss for n pulses.

m is noise pulses.

n is signal - to - noise pulses.

2.8.5 Propagation Effects

- The propagation effects of radar wave has significant impact on losses. Major effects of propagation on radar performance are under mentioned.
 1. Reflections from earth's surface
 2. Refraction
 3. Propagation in atmospheric ducts
 4. Attenuation in clear atmosphere.
- The propagation effects are not computed under system loss but under propagation factor.

The simple form of the Radar Equation

①

If the transmitter power P_t is radiated by an isotropic antenna (one that radiates uniformly in all directions), the power density (watt/Area) at a distance R from the radar is equal to the radiated power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius R .

$\therefore$ power density at Range R from an isotropic antenna $= \frac{P_t}{4\pi R^2}$

The power density at the target from a directive antenna with a transmitting Gain G is then

power density at Range R from a directive antenna $= \frac{P_t G}{4\pi R^2}$

The Radar cross section of the target determines the power density returned to the radar for a particular power density incident on the target. It is denoted by σ

Re-radiated power density back at the

$$\text{radar} = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2}$$

The radar antenna captures a portion of the echo energy incident on it. If the effective area of the receiving antenna is A_e and the power P_r received by radar is

$$P_r = \frac{P_t G}{4\pi R^2} \frac{\sigma}{4\pi R^2} A_e$$

$$= \frac{P_t G \sigma A_e}{(4\pi R^2)^2}$$

The maximum Radar range (R_{max}) is the distance beyond which the target can't be detected. It occurs when the received echo signal power P_r just equals the minimum detectable signal S_{min}

$$(R_{max})^4 = \frac{P_t G \sigma A_e}{(4\pi)^2 S_{min}}$$

$$R_{max} = \left(\frac{P_t G \sigma A_e}{(4\pi)^2 S_{min}} \right)^{1/4}$$

This eqⁿ is called fundamental radar eqⁿ.

$$* G = \frac{4\pi A_e}{\lambda^2} \quad A_e = \frac{G \lambda^2}{4\pi}$$

$$\therefore R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}} \right)^{1/4}$$

②
 → In a Radar system it is observed that the echo received is after $9.15 \mu\text{sec}$. Calculate the distance of the stationary object from the radar.

Sol

$$T_R = 9.15 \mu\text{sec}$$

$$\begin{aligned} R &= \frac{C T_R}{2} \\ &= \frac{3 \times 10^8 \text{ m/sec} \times 9.15 \times 10^{-6} \text{ sec}}{2} \\ &= 1.372 \text{ Km} \end{aligned}$$

→ A radar system transmits pulses of duration $2 \mu\text{s}$ and repetition rate of 1 KHz . Find the maximum and minimum range of radar.

Sol

$$\text{PRF } f_r = 1 \text{ KHz}$$

$$P.W = 2 \mu\text{sec}$$

The maximum Range of radar is

$$\begin{aligned} R_{\text{max}} &= \frac{C}{2 f_r} = \frac{3 \times 10^8 \text{ m/s}}{2 \times 1 \times 10^3} \\ &= 1.5 \times 10^5 \text{ m.} \end{aligned}$$

$$\begin{aligned} R_{\text{min}} &= \frac{C \cdot P.W}{2} = \frac{3 \times 10^8 \times 2 \times 10^{-6}}{2} \\ &= 300 \text{ meters.} \end{aligned}$$

→ what is the duty cycle of radar with a pulse width of $4 \mu\text{sec}$ and a pulse repetition time of 8 msec ?

Sol

$$\text{Duty cycle} = \frac{P.W}{P.R.T} = \frac{4 \times 10^{-6}}{8 \times 10^{-3}} = 0.0005.$$

→ A Radar is to have a maximum of 300km. What is the maximum allowable pulse repetition freq for unambiguous reception?

sol

The maximum unambiguous range is given by

$$R_{\max} = \frac{C}{2f_r}$$

$$f_r = \frac{C}{2R_{\max}}$$

$$= \frac{3 \times 10^8}{2 \times 300 \times 10^3}$$

$$= 500 \text{ pulse per sec}$$

$$= 0.5 \text{ KHz}$$

→ calculate the maximum range of radar which operates at a freq of 10GHz, Peak pulse power of 600KW. If the antenna effective area is 5m² and the area of target is 20m², minimum receivable power is 10⁻¹³ watt.

sol $P_t = 600 \text{ KW}$ $\sigma = 20 \text{ m}^2$ $A_e = 5 \text{ m}^2$ $P_{\min} = 10^{-13} \text{ w}$ (or) $S_{\min}$

$$f = 10 \times 10^9 \text{ Hz}$$

$$\lambda = \frac{C}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03 \text{ m}$$

$$G_t = \frac{4\pi A_e}{\lambda^2} = 69.813 \times 10^3$$

$$R_{\max} = \left(\frac{P_t G A_e \sigma}{(4\pi)^2 P_{\min}} \right)^{1/4}$$

$$= 717 \text{ Km}$$

→ A marine radar operating at 10 GHz has a max. range of 50 km with an antenna gain of 4000. If the transmitter has a power of 250 kW and minimum detectable signal of 10^{-11} W. Determine the cross-section of the target the radar can sight. ③

sol

$$f = 10 \times 10^9 \text{ Hz} \quad R_{\max} = 50 \text{ km}$$

$$P_t = 250 \text{ kW} \quad G_t = 4000$$

$$P_{\min} (S) S_{\min} = 10^{-11} \text{ W}$$

$$\lambda = \frac{c}{f} = 0.03 \text{ m}$$

$$\therefore R_{\max} = \left[\frac{P_t G_t^2 \lambda^2}{(4\pi)^3 S_{\min}} \right]^{1/4}$$

$$\sigma = \frac{R_{\max}^4 (4\pi)^3 S_{\min}}{P_t G_t^2 \lambda^2}$$

$$= 34.45 \text{ m}^2$$

→ An S-band radar transmitting at 3 GHz radiates 200 kW. Determine the signal power density at ranges 100 nautical miles if the effective area of the radar antenna is 9 m².

sol

$$f = 3 \text{ GHz} = 3 \times 10^9 \text{ Hz}$$

$$P_t = 200 \text{ kW} = 200 \times 10^3 \text{ watts}$$

$$R = 100 \text{ nautical miles}$$

$$1 \text{ nautical mile} = 1.8412 \times 10^3 \text{ m}$$

$$\therefore R = 1.8412 \times 10^5 \text{ meter}$$

$$A_e = 9 \text{ m}^2 \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m}$$

$$G_t = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \times 9}{(0.1)^2} = 11.3 \times 10^3$$

power density by directive antenna is

$$P = \frac{P_t G_t}{4\pi R^2} \\ = 5.248 \text{ mW/m}^2$$

→ A Radar operating at 3GHz radiating power of 200 kW. Calculate the power of the reflected signal at the radar with a 20m² target at 300 nautical miles. Take $A_e = 9 \text{ m}^2$

Sol

$$P_{sr} = \frac{P_t G_t A_e}{(4\pi)^2 R^4} \quad G_t = \frac{4\pi A_e}{\lambda^2} \\ R = 300 \times 1.852 \times 10^3 \text{ m} \\ = 27.034 \times 10^{15} \text{ watts}$$

→ Find the maximum range of a radar, the transmitted power is 250 kW, cross sectional area of the target is 12.5 m², minimum power received is 10⁻³ watt. receiver antenna gain is 2000 and operating wavelength 16cm.

Sol

$$R_{\max} = \left(\frac{P_t G_t \lambda^2}{(4\pi)^3 S_{\min}} \right)^{1/4} \\ = 200.39 \text{ km}$$

$G_r = G_t$
common antenna is used for transmission & reception
 $\therefore G_r = G_t$

→ A pulsed radar operating at 10GHz has an antenna with a gain of 28dB and a transmitting power of 2kW . If it is desired to detect a target with cross section of 12m^2 and the minimum detectable signal $S_{\min} = -90\text{dBm}$. what is the maximum range of radar.

Sol

$$f = 10 \times 10^9 \text{ Hz} \quad \lambda = \frac{c}{f} = 0.03 \text{ m}$$

$$G_t = 28 \text{ dB} = 630.95 \quad P_t = 2 \times 10^3 \text{ W}$$

$$\sigma = 12 \text{ m}^2 \quad P_{\min} \text{ (or)} S_{\min} = -90 \text{ dBm} = 10^{-12} \text{ watt}$$

$$\therefore R_{\max} = \left(\frac{P_t G_t A_e \lambda^2 \sigma}{(4\pi)^3 S_{\min}} \right)^{1/4}$$

$$= 1619 = 1.619 \text{ km.}$$

Limitation of Radar Range eqⁿ

$$R_{\max} = \left(\frac{P_t G_t A_e \sigma}{(4\pi)^2 S_{\min}} \right)^{1/4} \quad \text{--- (1)} \quad G_t = \frac{4\pi A_e}{\lambda^2}$$

$$R_{\max} = \left(\frac{P_t G_t^2 \lambda^2 \sigma}{(4\pi)^3 S_{\min}} \right)^{1/4} \quad \text{--- (2)} \quad A_e = \frac{G_t \lambda^2}{4\pi}$$

$$R_{\max} = \left(\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{\min}} \right)^{1/4} \quad \text{--- (3)}$$

These three forms of the radar eqⁿ are basically the same but there are diff in interpretation.

for example in eq (2)

$$R_{\max} \propto \lambda^{1/2}$$

& in eq (3)

$$R_{\max} \propto \lambda^{-1/2}$$

which is just opposite.

The correct interpretation depends on whether the antenna gain is held constant with change in wavelength or freq as implied by eq (2)

(81) the effective area is held constant for eq (3)

Receiver Noise

Noise is unwanted electromagnetic energy which interferes with the ability of the receiver to detect the wanted signal thus limiting the receiver sensitivity. It may originate within the receiver itself or it may enter via the receiving antenna along with the desired signal.

If the radar were to operate in a perfectly noise free environment so that no external noise accompany the target signal, & if the receiver itself were so perfect that it did not generate any excess noise, there would be still be noise generated by the thermal motion of the conduction electrons in the ohmic portion of the receiver

1/P statges. This is called thermal noise or Johnson noise. ⑥

The available thermal noise power generated by a receiver of B.W B_n (hertz) at a temperature T (°K) is

$$\text{thermal noise power} = KTB_n$$

$$K = \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J/deg.}$$

~~The~~ For the superhetrodyne receiver mostly used in radar, the receiver B.W is approximately that of the intermediate freq B.W. (IF amplifier)

$$B_n = \frac{\int_{-\infty}^{\infty} |H(f)|^2 df}{|H(f_0)|^2}$$

$H(f)$ = freq response funⁿ of the IF amplifier

f_0 = freq of maximum response.

The Bandwidth B_n is called the noise bandwidth is the band width of the equivalent rectangular filter whose noise power o/p is same as the filter with freq response funⁿ $H(f)$.

But noise band width is not theoretically same as the 3-dB (or) half power bandwidth.

3-dB BW is defined by the separation b/w the points of the freq response funⁿ $H(f)$ where the response is reduced 0.707 (3dB in power) from its maximum value.

Actually 3dB-bandwidth is used in many cases as an approximation to the noise B.W.

The measure of the noise out of a real receiver to that from the ideal receiver with only thermal noise is called the noise figure and is defined as

$$F_n = \frac{N_{out}}{K T_0 B_n G_a} = \frac{\text{noise out of practical receiver}}{\text{noise out of ideal receiver at std temp } T_0}$$

G_a = available gain

T_0 = 290 K std temp

The available gain G_a is the ratio of the signal S_{out} to the signal in S_{in} , with both o/p and i/p matched to deliver maximum o/p power.

The i/p noise N_{in} , in an ideal receiver is $K T_0 B_n$

$$\therefore \text{Noise figure } F_n = \frac{N_{out}}{\frac{S_{out}}{S_{in}} N_{in}} = \frac{S_{in}/N_{in}}{S_{out}/N_{out}}$$

$$\therefore \text{i/p signal } S_{in} = \frac{K T_0 B_n F_n S_o}{N_o}$$

If the minimum detectable signal S_{min} is that value of S_{in} , which corresponds to the min detectable signal to noise at the o/p of the IF $(S_{out}/N_{out})_{min}$.

$$S_{min} = K T_0 B_n F_n \left(\frac{S_{out}}{N_{out}} \right)_{min}$$

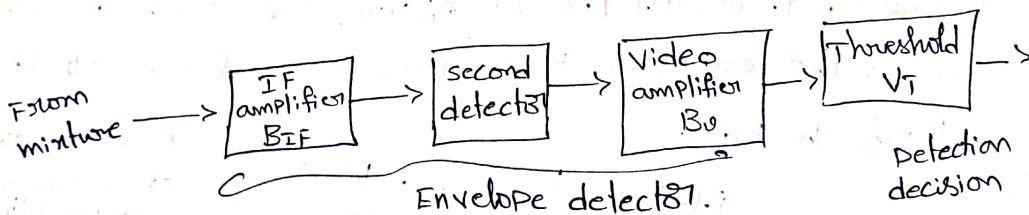
∴ the modified Radar eqⁿ is

$$R_{max}^4 = \frac{P_t G A_e \sigma}{(4\pi)^2 K T_0 B_n F_n (S/N_0)_{min}}$$

signal to noise Ratio

Signal to noise ratio is very important as far as radar is concerned. Because presence of target if not have small dist. statistical noise theory will be applied to obtain S/N_0 at the o/p of the IF amplifier necessary to achieve a specified prob of detection without exceeding a specified prob of false alarm.

The details of system that is considered



Consider

- IF amplifier with Band width B_{IF} followed by a second detector and a video amplifier with Band width B_v .
- the second detector and video amplifier are assumed to form an envelope detector, that is one which rejects the carrier freq. but passes the modulation envelope.

→ To extract the modulation envelope, the video BW must be wide enough to pass the low freq components generated by the second detector but not so wide as to pass the high freq components at or near the IF.

→ The video bandwidth B_v must be greater than $B_{IF}/2$ in order to pass all video modulation.

Probability of false alarm

The receiver noise at the input to the IF filter is described by the gaussian probability density f_u^n with mean value zero

$$P(v) = \frac{1}{\sqrt{2\pi}\psi_0} \exp \left(-\frac{v^2}{2\psi_0} \right)$$

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

S.O. Rice has shown that when the gaussian noise is passed through the IF filter, the prob density f_u^n of the envelope R is given by a form of the

Rayleigh Pdf.

$$P(R) = \frac{R}{\psi_0} \exp \left(-\frac{R^2}{2\psi_0} \right)$$

$R \rightarrow$ amplitude of envelope of the filter

The prob that the envelope of the noise voltage will exceed the voltage threshold V_T

$$\begin{aligned} P(V_T < R < \infty) &= \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp \left(-\frac{R^2}{2\psi_0} \right) dR \\ &= \exp \left(-\frac{V_T^2}{2\psi_0} \right) = P_{fa} \end{aligned}$$

whenever the voltage envelope exceeds the threshold V_T , a target is considered to have been detected, since the prob of a false alarm is the prob that noise will cross the threshold. ⑦

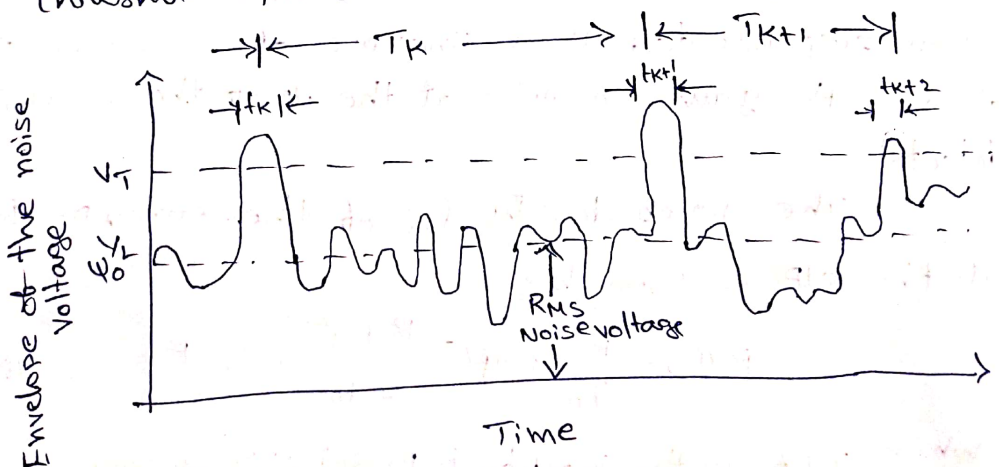
∴ prob of false alarm

$$P_{fa} = \exp\left(\frac{-V_T^2}{2\psi_0}\right)$$

The time b/w false alarms T_{fa} is also a better measure of the effect of noise on the radar performance. The average time b/w crossings of the threshold by noise alone is defined as the false alarm time T_{fa} .

$$T_{fa} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{K=1}^N T_K$$

T_K is the time b/w crossings of the threshold V_T by the noise envelope.



The false alarm probability may also be defined as the ratio of the duration of time the envelope is actually above the threshold to the total time it could have been above the threshold.

$$\text{i.e.; } P_{fa} = \frac{\sum_{k=1}^N t_k}{\sum_{k=1}^N T_k} = \frac{(t_k)_{av}}{(T_k)_{av}} = \frac{1}{T_{fa} B}$$

The avg duration of a threshold crossing by noise $(t_k)_{av}$ is approximately the reciprocal of the IF Bandwidth B . The avg of T_k is the false alarm time T_{fa} .

$$T_{fa} = \frac{1}{B} \exp\left(\frac{V_T^2}{2\psi_0}\right)$$

Prob of detection

so far we have discussed only the noise i/p at the radar receiver. Now consider an echo signal represented as a sine wave of amplitude A along with gaussian noise at the i/p of the envelope detector.

The prob density fun of the envelope R at the o/p is given by

$$P_s(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2 + A^2}{2\psi_0}\right) I_0\left(\frac{RA}{\psi_0}\right) \quad \text{--- (1)}$$

where $I_0(z)$ is the modified Bessel fun of zero order and argument z .

For large z , an asymptotic expansion for $I_0(z)$

is
$$I_0(z) = \frac{e^z}{\sqrt{2\pi z}} \left(1 + \frac{1}{8z} + \dots\right)$$

when the signal is absent $A=0$ eqn reduced

to then
$$P(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right)$$

The prob of detecting the signal is the prob that the envelope R will exceed the thresh. V_T .

$\therefore$ The prob of detection is

$$P_d = \int_{V_T}^{\infty} P_S(R) dR$$
$$= \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp\left[-\frac{(R^2 + A^2)}{2\psi_0}\right] I_0\left(\frac{RA}{\psi_0}\right) dR$$

In radar systems analysis it is more convenient to use signal to noise power ratio S/N than $A^2/2\psi_0$.

$$\frac{A^2}{\psi_0} = \frac{\text{signal amplitude}^2}{\text{rms noise voltage}^2} = \frac{\sqrt{2} (\text{rms signal voltage})^2}{2 \text{ m.s noise voltage}^2}$$
$$= 2 \left(\frac{\text{Signal power}}{\text{noise power}} \right) = \left(\frac{2S}{N} \right)$$

Integration of Radar Pulses

The no. of pulses returned after hitting target is given by

$$n = \frac{\theta_B f_p}{\theta_s} = \frac{\theta_B f_p}{\omega_r}$$

θ_B = antenna Beam width deg

f_p = pulse repetition freq Hz

θ_s = antenna scanning rate deg/sec

ω_r = revolutions per minute (rpm)

if 360° rotating antenna.

The process of summing all the radar echoes available from a target is called integration.

If the integration of pulses is performed before second detector in radar receiver is known as predetection integration or coherent integration. after second detector is called post detection integration or noncoherent integration.

→ If n pulses of same S/N were perfectly integrated by an ideal lossless predetection integrator, the integrated S/N will be exactly n times that of a single pulse

$$\therefore \left(\frac{S}{N}\right)_n = \frac{\left(\frac{S}{N}\right)_1}{n}$$

→ If n pulses of same S/N are integrated by post detection integration, the integrated S/N will be less than n times of single pulse. (9)

This loss in integration efficiency is caused by the non linear action of the second detector, which converts some of the signal energy to noise energy in the rectification process.

The integration efficiency for post-detection integration is given by

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n} \quad \text{--- (1)}$$

The improvement in S/N ratio when n pulses are integrated is called the "integration improvement factor" $I_i(n) = n E_i(n)$.

Marcum defined an integration loss in dB as $L_i(n) = 10 \log [1/E_i(n)]$.

The signal e_2^n with n pulses integrated can be written as

$$R_{\max}^4 = \frac{P_t G_r A_e}{(4\pi)^2 k T_0 B_n F_n (S/N)_n}$$

from eq (1)

$$R_{\max}^4 = \frac{P_t G_r A_e - n E_i(n)}{(4\pi)^2 k T_0 B_n F_n (S/N)_1}$$

Radar cross-section of targets

A radar cross section is defined as the ratio of its effective isotropic scattered power to the incident power density.

$$\sigma = \frac{\text{power reflected towards source / unit solid angle}}{\text{incident power density} / 4\pi}$$

$$= \lim_{R \rightarrow \infty} 4\pi R^2 \left| \frac{E_s}{E_i} \right|^2$$

R is the range of the target

E_s is the strength of reflected field at radar

E_i is the incident field at target

→ The radar cross-section depends on the characteristic dimensions of the object compared to the radar wavelength.

i) when the wavelength is large compared to the object's dimensions, scattering is said to be in Rayleigh region

ii) when the wavelength is small compared to the object's dimensions is called the optical region.

iii) In b/w the Rayleigh and optical regions is the resonance region where the radar wavelength is comparable to the object's dimensions

For many objects the radar cross section (10)
is larger in the resonance region than in
the other two regions.

Transmitter Power

$$R_{\max} = \left(\frac{P_t G A_e \sigma}{(4\pi)^2 S_{\min}} \right)^{1/4} \quad \text{--- (1)}$$

the power P_t in the eqn (1) is also called Peak Power of the pulse. The avg power P_{avg} of a radar is also an important measure of radar performance than the Peak Power. It is defined as the average transmitter power over the duration of the total transmission.

If the transmitter waveform is a train of rectangular pulses of width τ and constant pulse repetition period $T_p = 1/f_p$ then the avg power is related to Peak power by

$$P_{\text{avg}} = \frac{P_t \tau}{T_p} = P_t \tau f_p$$

$$\text{The radar duty cycle} = \frac{P_{\text{avg}}}{P_t} = \tau f_p$$

$$\therefore R_{\max}^4 = \frac{P_{\text{avg}} G A_e \sigma \eta E_i(n)}{(4\pi)^2 k T_0 F_n(B_n \tau) f_p (S/N)_1}$$

If the transmitted waveform is not rectangular pulse, sometimes it is more convenient to express radar eqⁿ in terms of energy

The energy per pulse

$$E_p = P_t T = \frac{P_{av}}{f_p}$$

$$\begin{aligned} \therefore R_{max} &= \frac{E_p G A_e \sim n E_i(n)}{(4\pi)^2 k T_0 F_n(B_n T) (S/N)}, \\ &= \frac{E_T G A_e \sim E_i(n)}{(4\pi)^2 k T_0 F_n(B_n T) (S/N)}, \end{aligned}$$

where E_T is the total energy of the 'n' pulses which equals $n E_p$ i.e; $E_T = n E_p$

The bandwidth and the pulse width are grouped together since the product of two is approximately unity in a well designed radar.

CW and Frequency Modulated Radar

CW and Frequency Modulated Radar

- Doppler Effect
 - CW Radar-Block Diagram
 - Isolation between Transmitter and Receiver
 - Non-zero IF receiver
 - Receiver Bandwidth Requirements
 - Applications of CW radar
 - Related Problems
-
- FM-CW radar
 - Range and Doppler measurement
 - Block diagram and Characteristics
 - FM-CW altimeter
 - Multiple Frequency CW radar

Doppler Effect

- Doppler effect implies that the frequency of a wave when transmitted by the source is not necessarily the same as the frequency of the transmitted wave when picked by the receiver.
- The received frequency depends upon the relative motion between the transmitter and receiver.
- If transmitter and receiver both are moving towards each other the received frequency higher, this is true even one is moving.
- If they are moving apart the received signal frequency decreases and if both are stationary, the frequency remains the same. This change in frequency is known as **Doppler shift**.
- Doppler shift depends upon the relative velocity between radar and target

- If **R** is the distance from the radar to target, the total number of wavelengths λ contained in the two-way path between the radar and the target are **$2R/\lambda$** .
- Each wavelength corresponds to a phase change of **2π** radians. The total phase change in the two way propagation path is then

$$\phi = 2\pi * \frac{2R}{\lambda} = 4\pi R/\lambda$$

- If the target is in motion relative to the radar, R is changing and so will the phase. Differentiating ϕ w.r.t time gives the rate of change of phase, which is the angular frequency

$$w_d = \frac{d\phi}{dt} = \frac{4\pi}{\lambda} \frac{dR}{dt} = \frac{4\pi v_r}{\lambda} = 2\pi f_d$$

- $v_r = \frac{dR}{dt}$ is the radial velocity or rate of change range with time
- $w_d = 2\pi f_d$ is the rate of change of ϕ with time is the angular frequency, where f_d is the Doppler frequency shift.

$$f_d = \frac{2v_r}{\lambda} = \frac{2fv_r}{c}$$

- The relative velocity may be written as $V_r = V \cos \theta$ where V is the target speed and θ is angle made by the target trajectory and the line joining radar and target. When $\theta=0$ the Doppler frequency is maximum. The Doppler is zero when the trajectory is perpendicular to the radar line of sight ($\theta=90^\circ$).

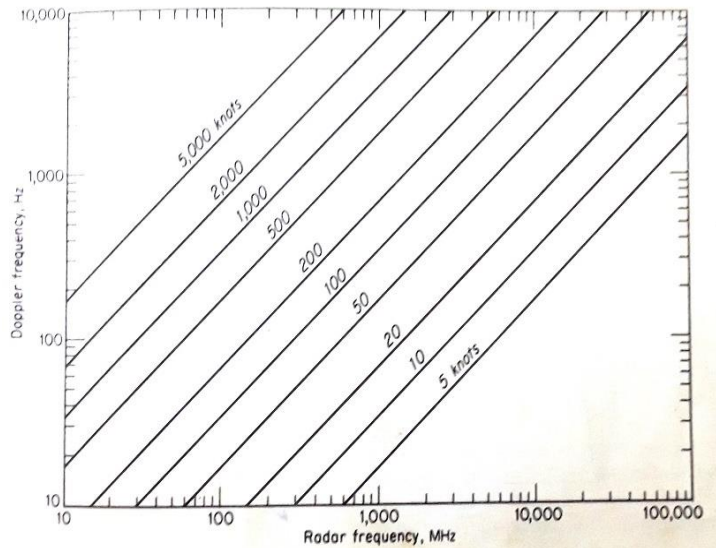
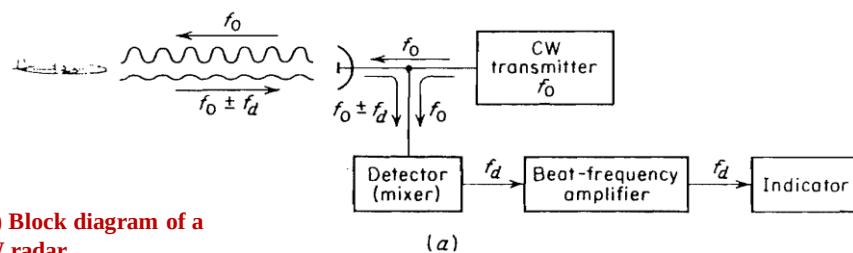


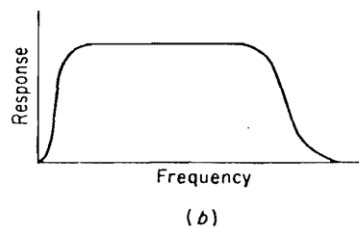
Figure: Doppler frequency f_d as a function of radar frequency and target relative velocity

CW RADAR

- It is possible to detect moving targets by radiating unmodulated Continuous wave energy instead of radiating in the form of pulses. Continuous Wave radars makes use of Doppler effect for target speed measurements.



**Figure : (a) Block diagram of a Simple CW radar
(b) Response characteristic of the doppler filter**



- Consider the simple CW radar shown the block diagram. The transmitter generates a continuous (unmodulated) oscillation of frequency f_0 which is radiated by the antenna.
- A portion of the radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna.
- If the target is in motion with a velocity V_r relative to the radar, the received signal will be shifted in frequency from the transmitted frequency f_0 by an amount $\pm f_d$ as given by the equation $f_d = \frac{2v_r}{\lambda} = \frac{2fv_r}{c}$
- *The plus sign associated with the Doppler frequency shift applies if the distance between target and radar is decreasing (approaching target) that is, when the received signal frequency is greater than the transmitted signal frequency. The minus sign applies if the distance is increasing (receding target).*
- The received echo signal at a frequency $f_0 \pm f_d$ enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal f_0 to produce a Doppler beat note of frequency f_d . The sign of f_d is lost in this process.
- The beat frequency amplifier eliminates the echoes from stationary targets and amplifies the Doppler echo signal.
- The low-frequency cutoff must be high enough to reject the d-c component caused by stationary targets, but yet it must be low enough to pass the smallest Doppler frequency expected. Sometimes both conditions cannot be met simultaneously and a compromise is necessary. The upper cutoff frequency is selected to pass the highest Doppler frequency expected.

Advantages of CW Radar

- CW Doppler radar has no blind speed.
- CW Doppler radar is capable of giving accurate measurements of relative velocities.
- CW Doppler radars are always on, they need low power and are compact in size.
- They can be used for small to large range with high degree of efficiency and accuracy.
- The performance of radar is not affected by stationary object.

Disadvantages of CW Doppler radar

- The maximum range of CW Doppler radar is limited by the power that radar can radiate.
- The target range can not be calculated by CW Doppler radar.
- There is possibility of ambiguous results when number of targets are more.

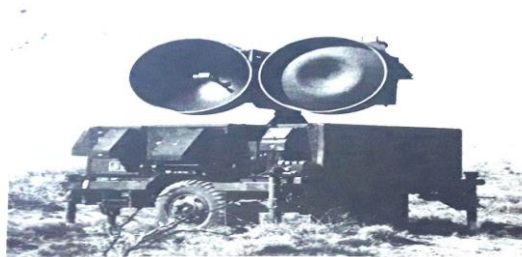
Applications of CW Radar

- CW Doppler radars are used where only velocity information is of interest and actual range is not needed. E.g: in LAW and Enforcement radar applications
- Measuring motion of wave on water level.
- Runway monitors.
- Cricket ball speed measurement.

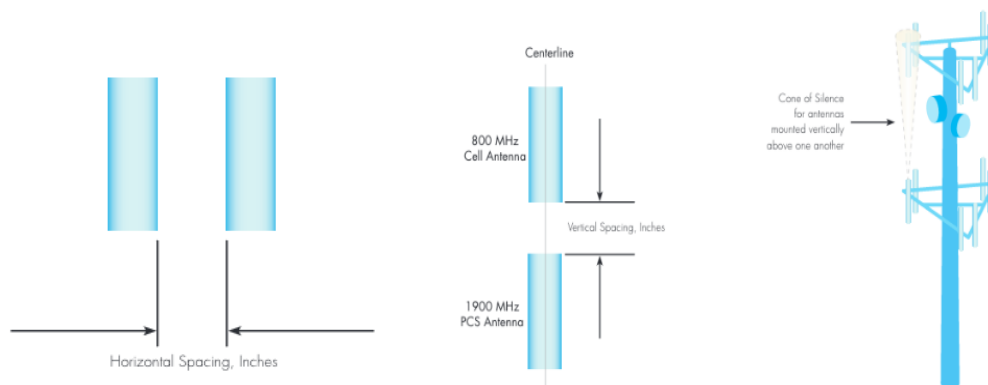
Isolation between transmitter and receiver

- A single antenna serves the purpose of both transmission and reception in the simple CW radar. In principle, a single antenna is sufficient as the necessary isolation is obtained by the separation in frequency (as a result of doppler effect), in practice there is considerable transmitter leakage.
- However, there are two reasons why the amount of transmitter leakage power should be kept at a low value.
 1. The maximum power the receiver input circuitry can withstand, without being physically damaged or having its sensitivity reduced, is quite low.
 2. The transmitter noise which enters the receiver from the transmitter reduces receiver sensitivity.
- The amount of isolation required depends on the transmitter power and the accompanying transmitter noise as well as the ruggedness and sensitivity of the receiver.
- *For example, If the safe value of power which might be applied to a receiver is 10mW and if the transmitter power is 1 kW, the isolation between transmitter and receiver must be at least 50 dB.*

- In long range CW applications, it is the level of the noise accompanying the transmitter leakage signal, rather than the damage this leakage might cause to the receiver circuitry, which determines the amount of isolation required.
- The amount of isolation which can be readily achieved between the arms of practical hybrid junctions such as the magic-T, rat race, or short-slot coupler is of the order of 20 to 30 dB.
- In some instances, when extreme precision is exercised, an isolation of perhaps 60 dB or more might be achieved. But one limitation of the hybrid junction is the 6-dB loss in overall performance.
- The largest isolations are obtained with two antennas one for transmission, the other for reception-physically separated from one another. Isolations of the order of 80 dB or more are possible with high-gain antennas. The more directive the antenna beam and the greater the spacing between antennas, the greater will be the isolation.
- The separate antennas of the AN/MPQ-46 CW tracker-illuminator of the Hawk missile system are shown



- The correct degree of isolation between transmitters and receivers can be implemented by one of two methods:
 - 1) Use two antennas, physically separated by a given distance, or;
 - 2) Use the appropriate duplexer with a single-antenna system.



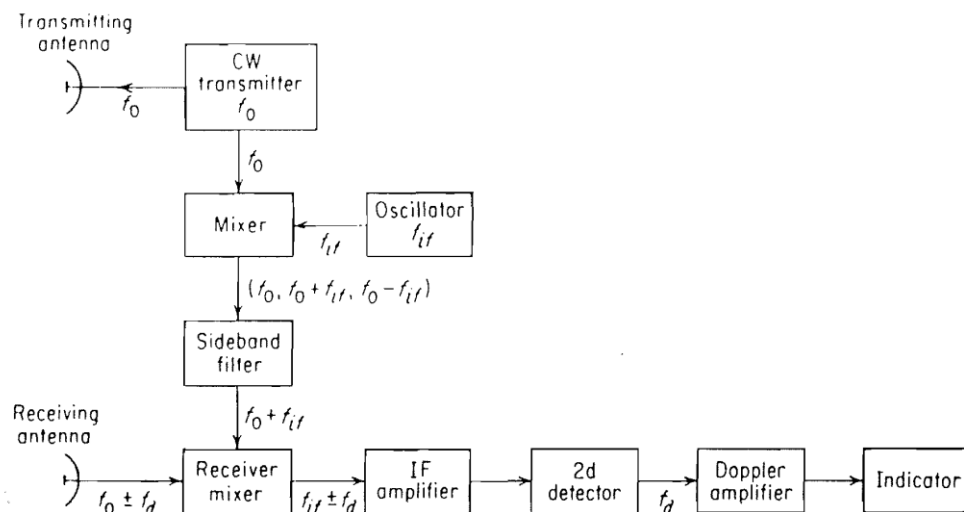
Intermediate-frequency receiver

Limitation of Zero IF receiver:

- Receivers of super heterodyne receiver type are also called homodyne receivers, or super heterodyne receivers with zero IF.
- However, this simpler receiver is not very sensitive because of increased noise at the lower intermediate frequencies caused by flicker effect.
- Flicker-effect noise occurs in semiconductor devices such as diode detectors and cathodes of vacuum tubes. The noise power produced by the flicker effect varies as $1/f^\alpha$ where α is approximately unity.
- At the lower range of frequencies (audio or video region), where the Doppler frequencies usually are found, the detector of the CW receiver can introduce a considerable amount of flicker noise, resulting in reduced receiver sensitivity.
- For short-range, low-power, applications this decrease in sensitivity might be tolerated
- But for maximum efficiency with CW radar, the reduction in sensitivity caused by the simple Doppler receiver with zero IF cannot be tolerated.

Non zero IF Receiver:

- Flicker effect noise reduces the receiver sensitivity of a CW Radar with zero IF (Simple Doppler radar). In order to increase the sensitivity and efficiency we go for CW Radar with Non-zero IF.



- Figure above shows the block diagram of a CW radar whose receiver operates with a nonzero IF. Separate antennas are shown for transmission and reception.
- Instead of the usual local oscillator found in the conventional super heterodyne receiver, the local oscillator (or reference signal) is derived in the receiver from a portion of the transmitted signal mixed with a locally generated signal of frequency equal to that of the receiver IF.
- Since the output of the mixer Consists of two sidebands on either side of the carrier plus higher harmonics, a narrow band filter selects one of the sidebands as the reference signal.
- The improvement in receiver sensitivity with an intermediate-frequency super heterodyne might be as much as 30 Db.

Limitations of CW radar with Non Zero IF

- False targets
- Unable to detect the range of the target

Receiver bandwidth requirements

- Bandwidth B , BW or Δf is the difference between the upper and lower cut-off frequencies of a radar receiver, and is typically measured in hertz.
- In case of a baseband channel or video signal, the bandwidth is equal to its upper cut-off frequency. In a Radar receiver the bandwidth is mostly determined by the IF filter stages.
- IF amplifier should be wide enough to pass the expected range of Doppler frequencies.
- Usually expected range of Doppler frequencies will be much higher than the frequency spectrum occupied by the signal energy . So a wide band amplifier is needed.
- which result in an increase in noise and a lowering of the receiver sensitivity and S/N.
- If the frequency of the Doppler-shifted echo signal are known beforehand, narrowband filter-that is just wide enough to reduce the excess noise without eliminating a significant amount of signal energy might be used.

- If the received waveform were a sine wave of infinite duration, its frequency spectrum would be a delta function as shown in the figure (a) below and the receiver bandwidth would be infinitesimal.
- But a sine wave of infinite duration and an infinitesimal bandwidth cannot occur in nature. The more normal situation is an echo signal which is a sine wave of finite duration.
- The frequency spectrum of a finite-duration sine wave has a shape of the form

$$\frac{\sin \pi(f-f_0)\delta}{\pi(f-f_0)}$$
 where f_0 and δ are the frequency and duration of the sine wave, respectively, and f is the frequency variable over which the spectrum is plotted (Fig b).

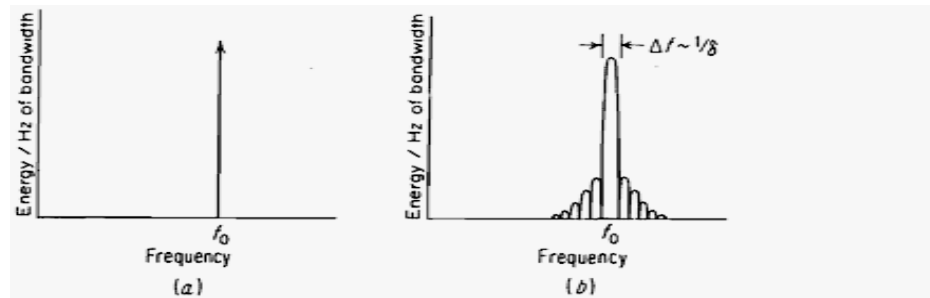


Figure 3.5 Frequency spectrum of CW oscillation of (a) infinite duration and (b) finite duration.

Filter Banks in CW radar Receiver

- A bank of narrowband filter is required to measure the frequency of echo signals. The filter bank also increases signal to noise ratio of radar receiver.

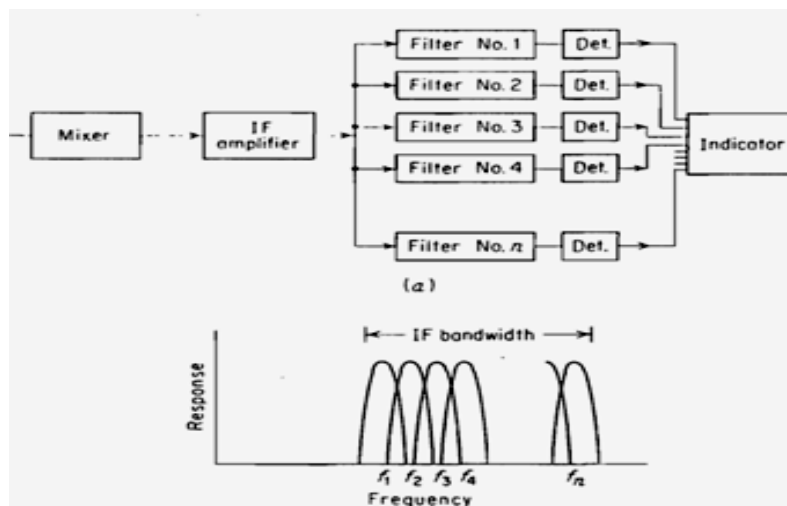


Figure: (a) Block diagram of IF Doppler filter bank (b) frequency-response characteristic of Doppler filter bank.

- BW of each filter is wide enough to accept the signal energy. But not so wide to accept the noise.
- The more the filters used less will be the SNR loss and less chance of missing a target.
- The ability to measure the magnitude of Doppler frequency and improvement in signal to noise ratio is better in IF filter bank than in video filter bank.
- Also the sign of Doppler shift (+ or -) is available which is not present in video filter bank.
- Each filter of filter bank has different bandwidth.

Sign of the radial velocity

- In many applications of CW radar it is of interest to know if the target is approaching or receding. This might be determined with separate filters located on either side of the intermediate frequency.
- If the echo-signal frequency lies below the carrier, then the target is receding; whereas if the echo frequency is greater than the carrier, then the target is approaching.

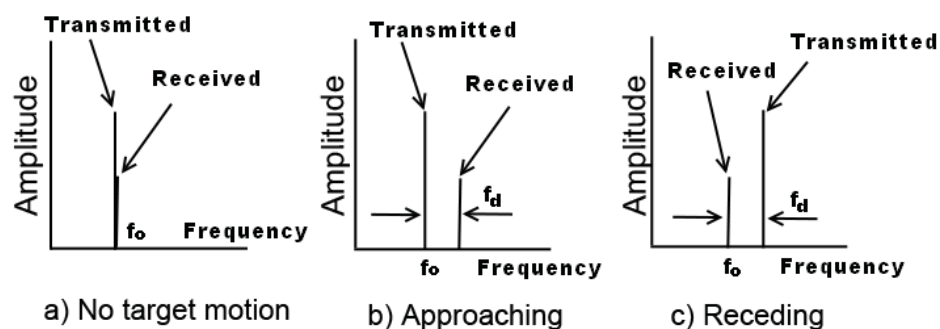


Figure: Spectra of received signals. (a) No Doppler shift, no relative target motion; (b) approaching target; (c) receding target.

- However, the Doppler-frequency spectrum "folds over" in the video because of the action of the detector, and hence the information about whether the doppler shift is positive or negative is lost. But it is possible to determine its sign from a technique borrowed from single-sideband communication.

- If the transmitter signal is given by,

$$E_t = E_0 \cos \omega_0 t$$

- The echo signal from the moving target will be,

$$E_r = K_1 E_0 \cos [(\omega_0 + \omega_d)t + \phi]$$

where, E_0 = amplitude of the transmitted signal

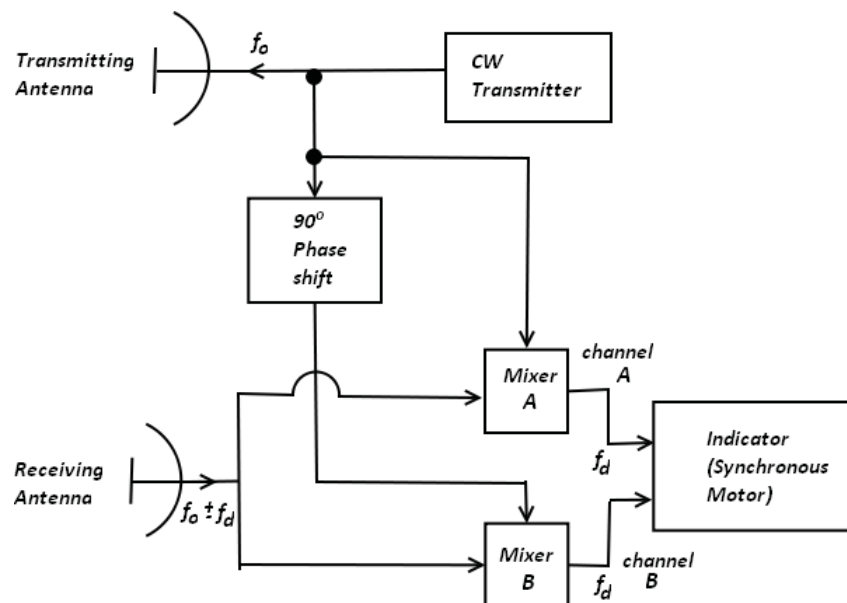
K_1 = a constant determined from the radar equation

ω_0 = angular frequency of transmitted signal, rad/sec

ω_d = dopper angular frequency shift, rad/sec

ϕ = a constant phase shift, which depends upon the range of initial detection (i.e., distance between the radar and the target)

- The sign of the Doppler frequency, and therefore the direction of target motion, may be found by splitting the received signal into two channels as shown



- In channel A the signal is processed as in a simple CW radar. The receiver signal and a portion of the transmitter signal heterodyne in the detector (mixer) to yield a difference signal,

$$E_A = K_2 E_0 \cos(\pm \omega t + \varphi)$$

- The channel B has $\pi/2$ phase delay introduced in the reference signal. The output of the channel B mixer is

$$E_B = K_2 E_0 \cos(\pm \omega t + \varphi + \pi/2)$$

- If the target is approaching (positive doppler), the outputs from the two channels are,

$$E_A = K_2 E_0 \cos(\omega t + \varphi)$$

$$E_B = K_2 E_0 \cos(\omega t + \varphi + \pi/2)$$

on the other hand, if the target is receding (negative doppler),

$$E_A(-) = K_2 E_0 \cos(\omega t - \varphi)$$

$$E_B(-) = K_2 E_0 \cos(\omega t - \varphi - \pi/2)$$

- The sign of ωd and the direction of the target's motion may be determined according to whether the output of channel B leads or lags the output of channel A.
- One method of determining the relative phase relationship between the two channels is to apply the outputs to a *synchronous two-phase motor*. The direction of motor rotation is an indication of the direction of the target motion.

Applications of CW radar

- Police speed monitor
- Rate-of-climb meter (During aircraft take off)
- Vehicle counting
- As a replacement for “5th wheel speedometer” in vehicle testing
- Antilock braking system
- Collision avoidance
- In railways as speedometer instead of tachometer
- Advance warning system for approaching targets
- Docking speed measurement of large ships
- Intruder alarms
- Measurement of velocity of missiles, baseball etc

FM CW RADAR

- FM CW radar is capable of measuring the relative velocity and the range of the target with the expense of bandwidth.
- The inability of the simple CW radar to measure range is related to the relatively narrow spectrum (bandwidth) of its transmitted waveform.
- By providing timing marks into the Tx signal the time of transmission and the time of return can be calculated. This will increase the bandwidth
- More distinct the timing, more accurate the result will be and more broader will the Tx spectrum
- The spectrum of a CW transmission can be broadened by the application of modulation, either amplitude, frequency, or phase.
- An example of an amplitude modulation is the pulse radar. The narrower the pulse, the more accurate the measurement of range and the broader the transmitted spectrum
- A widely used technique to broaden the spectrum of CW radar is **to frequency-modulate** the carrier. The timing mark is the changing frequency.

Range and Doppler measurement:

- In the frequency-modulated CW radar (abbreviated FM-CW), the transmitter frequency is changed as a function of time.
- Assume that the transmitter frequency increases linearly with time, as shown by the solid line in the next slide.
- If there is a reflecting object at a distance **R**, an echo signal will return after a time **$T = 2R/C$** . The dashed line in the figure represents the echo signal.
- If the echo signal is heterodyned with a portion of the transmitter signal in a nonlinear element such as a diode, a beat note ***fb will be produced***.
- If there is no Doppler frequency shift, the beat note (difference frequency) is a measure of the target's range and ***fb = fr where fr is the beat frequency due only*** to the target's range.

- If the rate of change of the carrier frequency is $\dot{f}_0$ (dot), then the beat frequency is given by

$$f_r = \dot{f}_0 T = \frac{2R}{c} \dot{f}_0$$

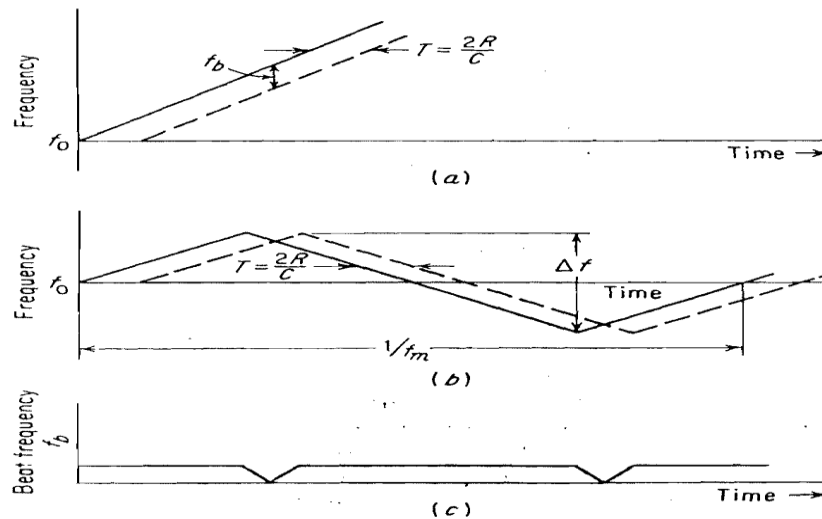


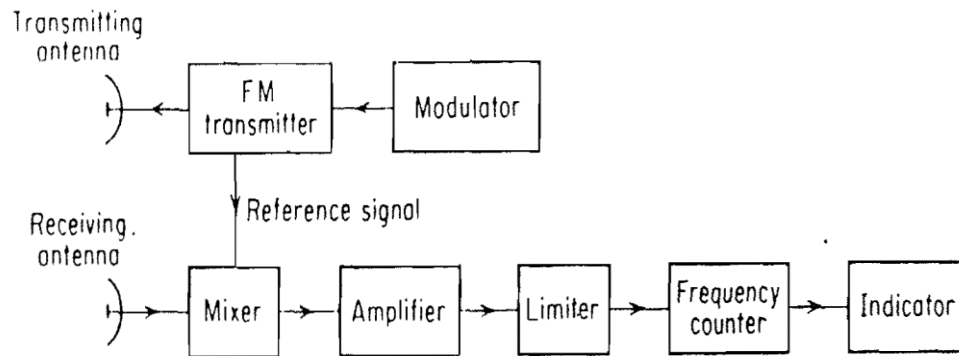
Figure: Frequency-time relationships in FM-CW radar. Solid curve represents transmitted signal; dashed curve represents echo. (a) Linear frequency modulation; (b)triangular frequency modulation; (c) beat note of (b).

- In any practical CW radar, the frequency cannot be continually changed in one direction. Periodicity in the modulation is necessary, as in the triangular frequency-modulation waveform shown in the figure (b). The modulation need not necessarily be triangular. It can be saw tooth, sinusoidal, or some other shape.
- The resulting beat frequency as a function of time is shown in figure(c) for triangular modulation.
- If the frequency is modulated at a rate f_m over a range Δf the beat frequency is

$$f_r = (2R/C).2f_m.\Delta f = 4Rf_m.\Delta f / C$$

- Thus the measurement of the beat frequency determines the range R.

Block diagram illustrating the principle of the FM-CW radar

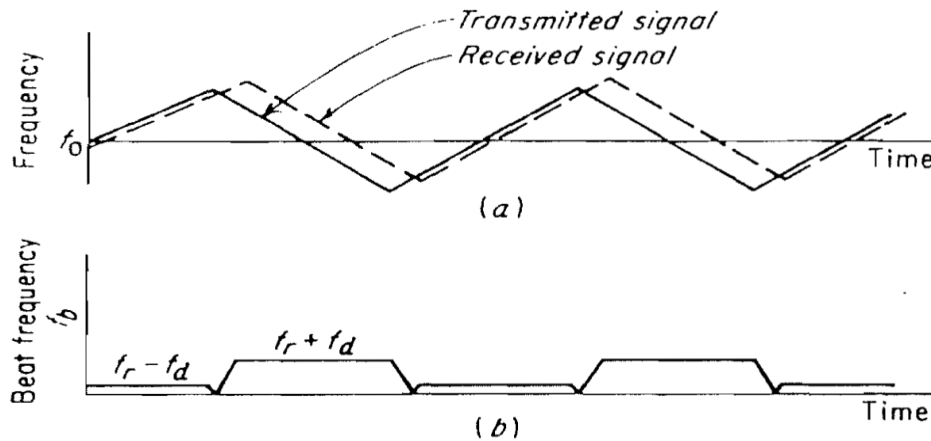


- A portion of the transmitter signal acts as the reference signal required to produce the beat frequency. It is introduced directly into the receiver via a cable or other direct connection.
- Ideally the isolation between transmitting and receiving antennas is made sufficiently large so as to reduce to a negligible level the transmitter leakage signal which arrives at the receiver via the coupling between antennas.
- The beat frequency is amplified and limited to remove any amplitude fluctuations.
- The frequency of the amplitude-limited beat note is measured with a cycle-counting frequency meter calibrated in distance.
- If the target is not stationary Doppler frequency shift will be superimposed on the FM range beat note and an erroneous range measurement results.
- The Doppler frequency shift causes the frequency-time plot of the echo signal to be shifted up or down as shown in the figure (a). On one portion of the frequency-modulation cycle, the beat frequency (Fig. b) is increased by the Doppler shift, while on the other portion, it is decreased.
- If for example, the target is approaching the radar, the beat frequency $f_b(\text{up})$ produced during the increasing or up will be the difference between the beat frequency due to the range f_r and the Doppler frequency shift f_d . Similarly, on the decreasing portion, the beat frequency $f_b(\text{down})$ is the sum of the two

$$f_b(\text{up}) = f_r - f_d$$

$$f_b(\text{down}) = f_r + f_d$$

Figure: Frequency-time relationships in FM-CW radar when the received signal is shifted in frequency by the Doppler effect (a) Transmitted (solid curve) and echo (dashed curve) (b) beat frequency



- The beat frequency due to range f_r can be calculated as

$$\frac{1}{2}[f_b(\text{up}) + f_b(\text{down})] = f_r$$

FM Altimeter

- The FM-CW radar principle is used in the aircraft radio altimeter to measure height above the surface of the earth.
- Relatively short ranges of altimeters permit Low Tx power and low antenna gain.
- Since the relative motion between the aircraft and ground is small, the effect of the Doppler frequency shift may usually be neglected.
- Frequency range: 4.2 to 4.4 GHz (reserved for altimeters)
- Solid state Tx is used here.
- High sensitive super-heterodyne Rx is preferred for better sensitivity and stability

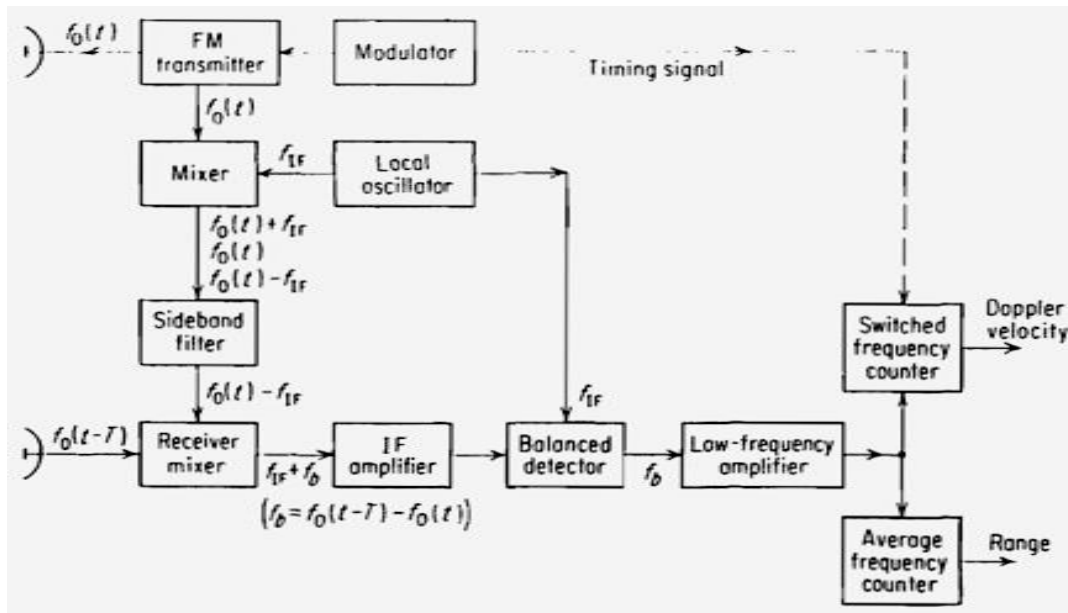


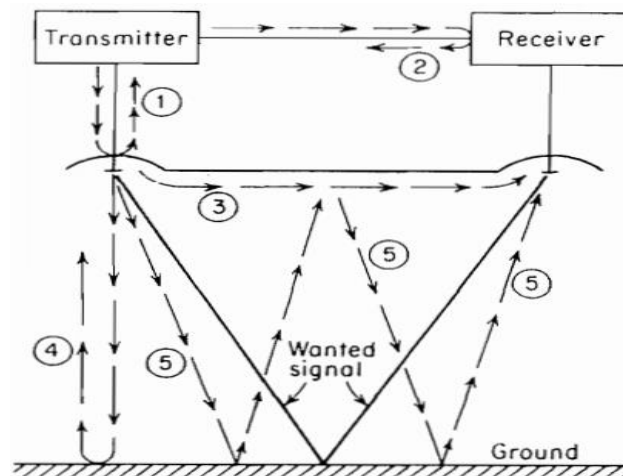
Figure: Block diagram of a FM-CW radar using sideband super heterodyne receiver

- A portion of the frequency-modulated transmitted signal is applied to a mixer along with the oscillator signal.
- The selection of the local-oscillator frequency is a bit different from that in the usual super heterodyne receiver. The local-oscillator frequency f_{IF} is the same as the intermediate frequency used in the receiver.
- The output of the mixer consists of the varying transmitter frequency $f_0(t)$ plus two sideband frequencies, one on either side of $f_0(t)$ and separated from $f_0(t)$ by the local-oscillator frequency f_{IF} .
- The filter selects the lower sideband, $f_0(t) - f_{IF}$ and rejects the carrier and the upper sideband. The sideband filter must have sufficient bandwidth to pass the modulation, but not the carrier or other sideband. The filtered sideband serves the function of the local oscillator.
- When an echo signal is present, the output of the receiver mixer is an IF signal of frequency $(f_{IF} + f_b)$ where f_b is composed of the range frequency f_r and the Doppler velocity frequency f_d .
- The IF signal is amplified and applied to the balanced detector along with the local-oscillator signal f_{IF} .
- The output of the detector contains the beat frequency (range frequency and the Doppler velocity frequency), which is amplified to a level where it can actuate the frequency-measuring circuits.

- The output of the low-frequency amplifier is divided into two channels: one feeds an average-frequency counter to determine the range, and the other feeds a switched frequency counter to determine the Doppler velocity (assuming $f_r > f_d$).
- Only the averaging frequency counter need be used in an altimeter application.
- A target at short range will generally result in a strong signal at low frequency, while one at long range will result in a weak signal at high frequency. Therefore the frequency characteristic of the low frequency amplifier in the FM-CW radar may be used to provide attenuation at the low frequencies corresponding to short ranges and large echo signals. Less attenuation is applied to the higher frequencies, where the echo signals are weaker.

• **Unwanted signals in FM altimeter:**

1. The reflection of the transmitted signals at the antenna caused by impedance mismatch.
2. The standing-wave pattern on the cable feeding the reference signal to the receiver, due to poor mixer match.
3. The leakage signal entering the receiver via coupling between transmitter and receiver antennas. This can limit the ultimate receiver sensitivity, especially at high altitudes.
3. The interference due to power being reflected back to the transmitter, causing a change in the impedance seen by the transmitter. This is usually important only at low altitudes. It can be reduced by an attenuator introduced in the transmission line at low altitude or by a directional coupler or an isolator.
4. The double-bounce signal.



Advantages of FM-CW Radar

- Range can be measured by simple broadening of frequency spectrum.
- FM modulation is easy to generate than linear modulation.
- Synchronization is not required as in multiple frequency CW radar.
- For measuring range, single frequency is required.

Disadvantages of FM-CW Radar

- FM CW radar can be used to detect single targets only.
- Accuracy of FM CW radar is less compared to Multiple frequency radar.
- Measurement of range is more difficult when FM signal is non uniform or mixer is not operating in linear region.

Multiple-frequency CW Radar

- Consider a CW radar radiating a single-frequency sine wave of the form $\sin 2\pi f_0 t$
- The signal travels to the target at a range R and returns to the radar after a time $T = 2R/c$ where c is the velocity of propagation.
- The echo signal received at the radar is $\sin [2\pi f_0(t-T)]$.
- If the transmitted and received signals are compared in a phase detector, the output is proportional to the phase difference between the two and is given by :

$$\Delta\phi = 2\pi f_0 T = 4\pi f_0 R/c.$$

- The phase difference may therefore be used as a measure of the range, or

$$R = \frac{c \Delta\phi}{4\pi f_0} = \frac{\lambda}{4\pi} \Delta\phi$$

- However, the measurement of the phase difference $\Delta\phi$ is unambiguous only if $\Delta\phi$ does not exceed 2π radians. Substituting $\Delta\phi = 2\pi$ into above eqn gives the maximum unambiguous range as $\lambda/2$.

- Unambiguous range may be extended considerably by utilizing two separate CW signals differing slightly in frequency.
- The transmitted waveform is assumed to consist of two continuous sine waves of frequency f_1 and f_2 separated by an amount Δf .
- For convenience, the amplitudes of all signals are set equal to unity.
- The voltage waveforms of the two components of the transmitted signal v_{1T} and v_{2T} may be written as:

$$v_{1T} = \sin(2\pi f_1 t + \phi_1)$$

$$v_{2T} = \sin(2\pi f_2 t + \phi_2)$$

where ϕ_1 and ϕ_2 are arbitrary (constant) phase angles.

- The echo signal is shifted in frequency by the Doppler effect. The form of the Doppler-shifted signals corresponding to the two frequencies f_1 and f_2 are:

$$v_{1R} = \sin \left[2\pi(f_1 \pm f_{d1})t - \frac{4\pi f_1 R_0}{c} + \phi_1 \right]$$

$$v_{2R} = \sin \left[2\pi(f_2 \pm f_{d2})t - \frac{4\pi f_2 R_0}{c} + \phi_2 \right]$$

Where R_0 = range to target at a particular time $t=t_0$

f_{d1} = Doppler frequency shift associated with frequency f_1

f_{d2} = Doppler frequency shift associated with frequency f_2

- Since the two RF frequencies f_1 and f_2 are approximately the same (that is $f_2 = f_1 + \Delta f$, where $\Delta f \ll f_1$) the Doppler frequency shifts f_{d1} and f_{d2} can be assumed to be equal to each other.
- Therefore we may write $f_{d1} = f_{d2} = f_d$
- The receiver separates the two components of the echo signal and heterodynes each received signal component with the corresponding transmitted waveform and extracts the two Doppler-frequency components given below:

$$v_{1D} = \sin \left(\pm 2\pi f_d t - \frac{4\pi f_1 R_0}{c} \right)$$

$$v_{2D} = \sin \left(\pm 2\pi f_d t - \frac{4\pi f_2 R_0}{c} \right)$$

- The phase difference between these two components is

$$\Delta\phi = \frac{4\pi(f_2 - f_1)R_0}{c} = \frac{4\pi \Delta f R_0}{c}$$

Hence

$$R_0 = \frac{c \Delta\phi}{4\pi \Delta f}$$

- which is the same as that of R in 37th slide with Δf substituted in place of f_0 .
- The two-frequency CW technique for measuring range was described as using the Doppler frequency shift.
- A large difference in frequency between the two transmitted signals improves the accuracy of the range measurement since large Δf means a proportionately large change in $\Delta\phi$ for a given range.
- However, there is a limit to the value of Δf since $\Delta\phi$ cannot be greater than 2π radians if the range is to remain unambiguous.
- The maximum unambiguous range R_{unamb} is:

$$R_{unamb} = \frac{c}{2 \Delta f}$$

- Therefore Δf must be less than $c/2R_{unamb}$. Note that when Δf is replaced by the pulse repetition rate, above eq gives the maximum unambiguous range of a pulse radar.
- The two-frequency CW radar is essentially a single-target radar since only one phase difference can be measured at a time. If more than one target is present, the echo signal becomes complicated and the meaning of the phase measurement becomes doubtful.
- The theoretical accuracy with which range can be measured with the two-frequency CW radar can be found and it can be shown that the theoretical rms range error is

$$\delta R = \frac{c}{4\pi \Delta f (2E/N_0)^{1/2}}$$

- Where E = energy contained in received signal and
- N_0 = noise power per hertz of bandwidth.

The above Equation indicates that the greater the separation Δf between the two frequencies, the lesser will be the **rms** error.

- Therefore the frequency difference must not be too large if unambiguous measurements are to be made.
 - The selection of Δf represents a compromise between the requirements of accuracy and ambiguity.
 - Both accurate and unambiguous range measurements can be made by transmitting three or more frequencies instead of just two.
 - For example, if the three frequencies f_1, f_2 and f_3 are such that $f_3 - f_1 = k(f_2 - f_1)$ where k is a factor of the order of 10 or 20, the pair of frequencies f_3, f_1 (with greater Δf) gives an ambiguous but accurate range measurement while the pair of frequencies f_2, f_1 (with lesser Δf) resolve the ambiguities in the measurement of Range.
 - Likewise, if further accuracy is required a fourth frequency can be transmitted and its ambiguities resolved by the less accurate but higher unambiguous measurement obtained from the three frequencies, f_1, f_2 and f_3 .
 - As more frequencies are added the spectrum and target resolution approach that obtained with a pulse or an **FM-CW** waveform.
-
- The multiple-frequency CW radar technique has been applied to the accurate measurement of distance in surveying and in missile guidance. The ***Tellurometer*** is the name given to a portable electronic surveying instrument which is based on this principle.
 - In addition to its use in surveying, the multiple CW frequency method of measuring range has been applied in
 1. Range-instrumentation radar for the measurement of the distance to a transponder-equipped missile,
 2. The distance to satellites,
 3. In satellite navigation systems based on range measurement
 4. For detecting the presence of an obstacle in the path of a moving automobile by measuring the distance,
 5. The Doppler velocity, and the sign of the Doppler (whether the target is approaching or receding).

- **Example1: Determine the Range and Doppler velocity of an approaching target using a triangular modulation FMCW Radar. Given : Beat frequency fb(up) = 15KHz and fb (down) = 25KHz , modulating frequency : 1MHz, Δf : 1KHz and Operating frequency : 3Ghz**

Solution:

We know $f_r = \frac{1}{2}[f_b(\text{up}) + f_b(\text{down})] = \frac{1}{2}(15+25) = 20 \text{ KHz}$

$f_d = \frac{1}{2}[f_b(\text{down}) - f_b(\text{up})] = \frac{1}{2}(25-15) = 5 \text{ KHz}$

The Range R in terms of f_r , f_m and Δf is given by : $R = c f_r / 4 f_m \Delta f =$
 $(3 \times 10^8) 20 \times 10^3 / 4(1 \times 10^6 \times 1 \times 10^3) \text{ mtrs} = 1500 \text{ mtrs} = 1.5 \text{ Kms}$

MTI AND PULSE DOPPLER RADAR

Introduction

Principle

MTI Radar with - Power Amplifier Transmitter and Power Oscillator
Transmitter

Delay Line Cancellers – Filter Characteristics

Blind Speeds

Double Cancellation

Staggered PRFs

Range Gated Doppler Filters

MTI Radar Parameters

Limitations to MTI Performance

MTI versus Pulse Doppler Radar.

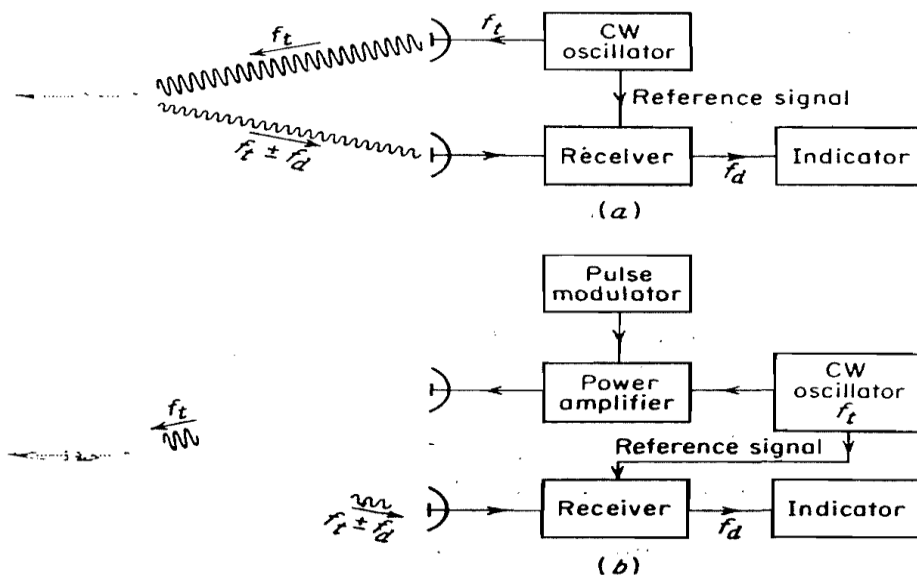
Introduction

- The Doppler frequency shift [$f_d = 2V_r / \lambda$] produced by a moving target also used in a pulse radar just as in the CW radar, to determine the relative velocity of a target or to separate desired moving targets from undesired stationary objects (clutter).
 - Pulse radar that utilizes the Doppler frequency shift as a means of discriminating moving targets from fixed targets is called a **MTI** (moving target indication) or a **pulse Doppler** radar.
 - The two are based on the same physical principle, but in practice there are differences between **MTI** and **Pulse Doppler** radar.
 - The MTI radar, usually operates with ambiguous Doppler measurement (so-called **blind speeds**) but with unambiguous range measurement (no second-time around echoes).
 - A pulse Doppler radar operates with ambiguous range measurement but with unambiguous Doppler measurement. Its pulse repetition frequency is usually high enough to operate with unambiguous Doppler (no Blind speeds) but at the expense of range ambiguities.
 - The discussion in this chapter, mostly is based on the **MTI Radar**, but much of what applies to MTI can be extended to **Pulse Doppler Radar** as well.
-
- **MTI** is a necessity in high-quality air-surveillance radars that operate in the presence of clutter.
 - Its design is more challenging than that of a simple pulse radar or a simple CW radar.
 - A **MTI** capability adds to a radar's cost and complexity.
 - The basic MTI concepts were introduced during World War 2, and most of the signal processing theory on which **MTI** (and **pulse Doppler**) radar depends was formulated during the mid-1950s.
 - However, the implementation of theory to practice was speeded up only subsequently after the availability of the necessary signal-processing technology.
 - It took almost twenty years for the full capabilities offered by MTI signal-processing theory to be converted into practical and economical Radar equipment.
 - The chief factor that made this possible was the development of reliable, small, and inexpensive digital processing hardware.

Principle of Operation

- A simple CW radar studied earlier is shown in Fig.1(a). It consists of a transmitter, receiver, indicator, and the necessary antennas.
- In principle, the CW radar converted into a pulse radar as shown in Fig.1(b) by providing a power amplifier and a modulator to turn the amplifier on and off for the purpose of generating pulses.
- The chief difference between the pulse radar of Fig. 1(b) and the one studied earlier is that a small portion of the CW oscillator power that generates the transmitted pulses is diverted to the receiver to take the place of the local oscillator.
- this CW signal also acts as the **coherent reference** needed to detect the Doppler frequency shift.
- By **coherent** it means that the phase of the transmitted signal is preserved in the reference signal.
- The reference signal is the distinguishing feature of **coherent MTI radar**.

Figure 1: (a) Simple CW Radar (b) Pulse Radar using Doppler Information



If the CW oscillator voltage is represented as $A_1 \sin 2\pi f_t t$ where A_1 is the amplitude and f_t the carrier frequency

Then the reference signal is: $V_{\text{ref}} = A_2 \sin 2\pi f_t t$ (1)

- And the Doppler-shifted echo-signal voltage is:

$$V_{\text{echo}} = A_3 \sin \left[2\pi(f_t \pm f_d)t - \frac{4\pi f_t R_0}{c} \right] \quad \text{.....(2)}$$

Where A_2 = amplitude of reference signal

A_3 = amplitude of signal received from a target at a range R_0

f_d = Doppler frequency shift

t = time

c = velocity of propagation

The reference signal and the target echo signal are heterodyned in the mixer stage of the receiver. Only the low-frequency (difference-frequency) component from the mixer is of interest and is a voltage given by:

$$V_{\text{diff}} = A_4 \sin \left(2\pi f_d t - \frac{4\pi f_t R_0}{c} \right) \quad \text{.....(3)}$$

- Note that the equations **(1) to (3)** above represent sine wave carriers upon which the pulse modulation is imposed.
- For stationary targets the Doppler frequency shift will be zero and hence V_{diff} will not vary with time and may take on any constant value from $+A_4$ to $-A_4$ including zero.
- However, when the target is in motion relative to the radar f_d has a value other than zero and the voltage corresponding to the difference frequency from the mixer **[Eq. (3)]** will be a function of time.
- An example of the output from the mixer when the Doppler frequency f_d is large compared with the reciprocal of the pulse width is shown in **Fig.2(b)**.
- If, on the other hand f_d is small compared with the reciprocal of the pulse duration, the pulses will be modulated with an amplitude given by Eq. **(3)** **[Fig. 2(c)]** and many pulses will be needed to extract the Doppler information.
- The case illustrated in **Fig. 2(c)** is more typical of aircraft-detection radar, while the waveform of **Fig. 2(b)** might be more applicable to a radar used for the detection of extraterrestrial targets such as ballistic missiles or satellites.

- The video signals shown in Fig.2 are called **bipolar**, since they contain both positive and negative amplitudes.

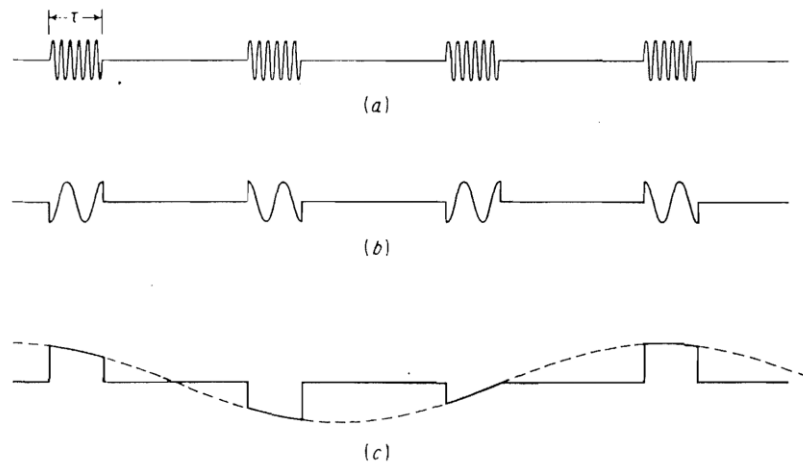
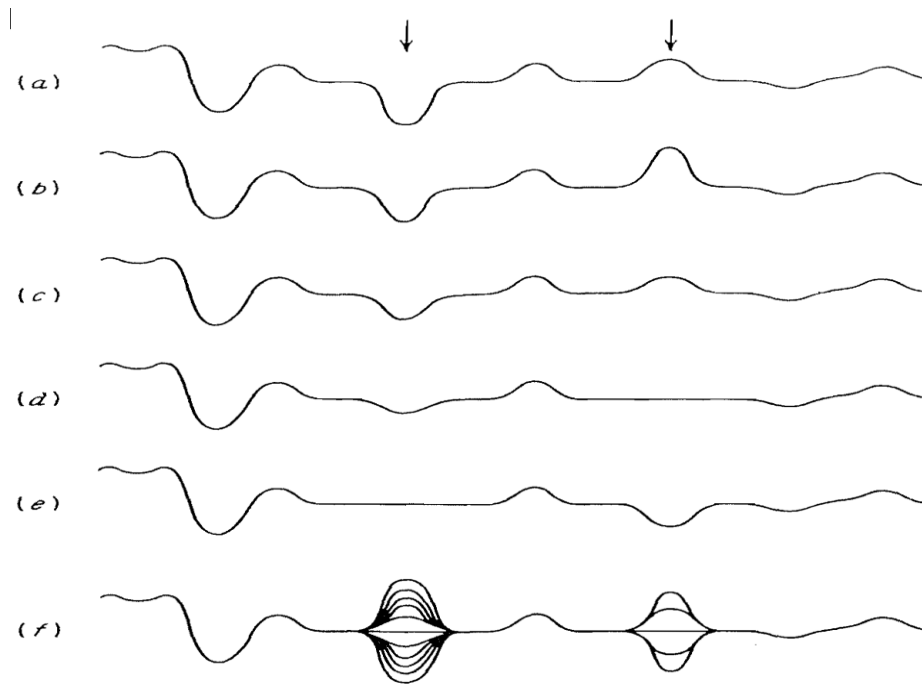


Figure 2 (a) RF or IF echo pulse train (b) video pulse train for Doppler frequency $f_d > 1/\tau$ (c) video pulse train for Doppler frequency $f_d < 1/\tau$.

- Moving targets may be distinguished from stationary targets by observing the video output on an A-scope (amplitude vs. range).
- A single sweep on an A-scope might appear as in Fig. 3 (a) shown below. This sweep shows several fixed targets and two moving targets indicated by the two arrows.
- On the basis of a single sweep, moving targets cannot be distinguished from fixed targets.
- Successive A-scope sweeps (pulse-repetition intervals) are shown in Fig. 3 (a) to (e).
- Echoes from fixed targets remain constant throughout, but echoes from moving targets vary in amplitude from sweep to sweep at a rate corresponding to the Doppler frequency.
- The superposition of the successive A-scope sweeps is shown in Fig. 3(f). The moving targets produce, with time, a "butterfly" effect on the A-scope.



**Figure 3 (a-e) Successive sweeps of a MTI radar A-scope display (echo amplitude as a function of time)
(f) superposition of many sweeps : arrows indicate position of moving targets.**

Delay-line canceler

- Although the butterfly effect is suitable for recognizing moving targets on an A-scope, it is not appropriate for display on the PPI. One method commonly employed to extract Doppler information in a form suitable for display on the PPI scope is with a delay-line canceler.
- The delay-line canceler acts as a filter to eliminate the d-c component of fixed targets and to pass the a-c components of moving targets.
- The video portion of the receiver is divided into two channels. One is a normal video channel. In the other, the video signal experiences a time delay equal to one pulse-repetition period (equal to the reciprocal of the pulse repetition frequency).
- The outputs from the two channels are subtracted from one another.
- The fixed targets with unchanging amplitudes from pulse to pulse are canceled on subtraction. However, the amplitudes of the moving-target echoes are not constant from pulse to pulse and subtraction results in an uncanceled residue.

- The output of the subtraction circuit is a bipolar video just as was the input.
- Before bipolar video can intensity-modulate a PPI display it must be converted into unipolar voltages (unipolar video) by a full-wave rectifier.

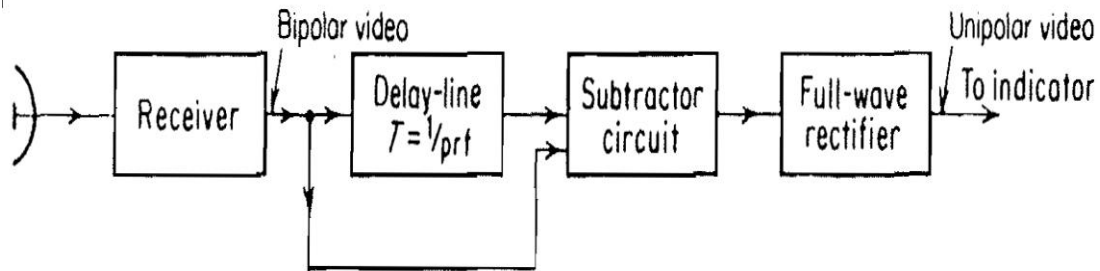
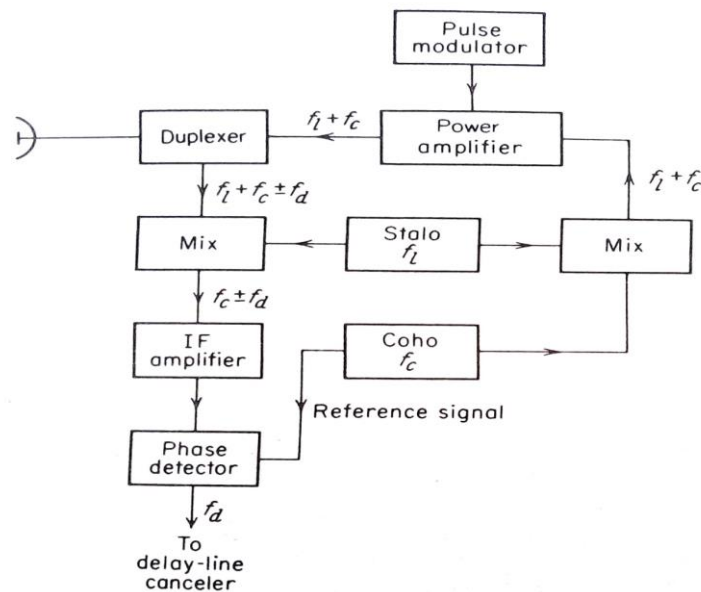


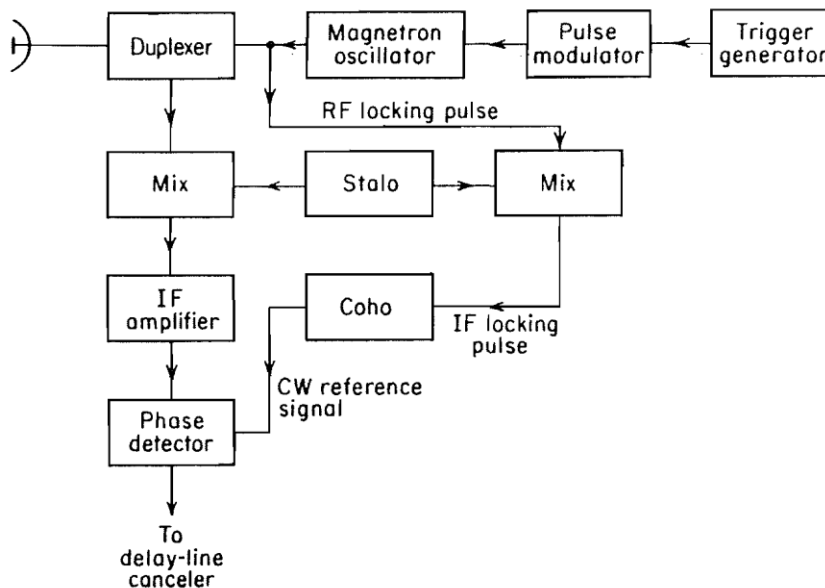
Figure 4 : MTI Receiver with delay-line canceler

MTI Radar with Power Amplifier



- The simple MTI radar shown in Fig. 1(b) is not the most typical. The block diagram of a more common MTI radar employing a power amplifier is shown above.
 - The significant difference between this MTI configuration and that of **Fig. 1(b)** is the manner in which the reference signal is generated.
 - In this diagram, the coherent reference is supplied by an oscillator called the **coho**, which stands for coherent oscillator.
 - The **coho** is a stable oscillator whose frequency is the same as the intermediate frequency used in the receiver.
 - In addition to providing the reference signal, the output of the **coho** is also mixed with the local-oscillator frequency f_l . This local oscillator also must be a stable oscillator and is called **stalo**, stands for stable local oscillator.
 - The RF echo signal is heterodyned with the stalo signal to produce the IF just as in the conventional super heterodyne receiver.
 - The **stalo**, **coho** and the mixer in which they are mixed are called Receiver- Exciter because of the dual role they serve both the receiver and the transmitter.
-
- The phase of the **stalo** influences the phase of the transmitted signal, any **stalo** phase shift is canceled on reception because the **stalo** that generates the transmitted signal also acts as the local oscillator in the receiver.
 - The reference signal from the **coho** and the IF echo signal are both fed into a mixer called the **Phase detector**.
 - The phase detector differs from the normal amplitude detector since its output is proportional to the phase difference between the two input signals.
 - Any one of a number of transmitting-tube types might be used as the power amplifier. These include the triode, tetrode, klystron, traveling-wave tube, and the crossed-field amplifier.
 - A transmitter which consists of a stable low-power oscillator followed by a power amplifier is sometimes called **MOPA**, which stands for **Master-Oscillator Power Amplifier**.

MTI Radar with Power Oscillator Transmitter



- Before the development of the klystron amplifier, the only high-power transmitter available at microwave frequencies for radar application was the magnetron oscillator.
- In an oscillator, the phase of the RF bears no relationship from pulse to pulse. For this reason, the reference signal cannot be generated by a continuously running oscillator.
- However, a coherent reference signal may be readily obtained with the power oscillator by readjusting the phase of the **cohо** at the beginning of each sweep according to the phase of the transmitted pulse.
- The phase of the **cohо** is locked to the phase of the transmitted pulse each time a pulse is generated.
- A block diagram of an MTI radar (with a power oscillator) is shown in below slide.
- A portion of the transmitted signal is mixed with the **stalo** output to produce an **IF** beat signal whose phase is directly related to the phase of the transmitter.
- This IF pulse is applied to the **cohо** and causes the phase of the **cohо** CW oscillation to "lock" in step with the phase of the IF reference pulse.
- The phase of the **cohо** is then related to the phase of the transmitted phase and may be used as the reference signal for echoes received from that particular transmitted pulse.
- Upon the next transmission, another **IF** locking pulse is generated to relock the phase of the CW **cohо** until the next locking pulse comes along.

Delay-line canceler

- The simple MTI delay-line canceller shown earlier is an example of a time-domain filter.
 - The capability of this device depends on the quality of the medium used as the delay line. The delay line must introduce a time delay equal to the pulse repetition interval.
 - For typical ground-based air surveillance radars this will be several milliseconds.
 - Delay times of this magnitude cannot be achieved with practical electromagnetic transmission lines.
 - By converting the electromagnetic signal to an acoustic signal, it is possible to utilize delay lines of a reasonable physical length since the velocity of propagation of acoustic waves is about 10^{-5} that of electromagnetic waves.
 - After the necessary delay is introduced by the acoustic line, the signal is converted back to an electromagnetic signal for further processing.
 - The early acoustic delay lines developed during World War 2 used liquid delay lines filled with either water or mercury. Liquid delay lines were large and inconvenient to use. They were replaced in the mid-1950s by the solid fused-quartz delay line that used multiple internal reflections to obtain a compact device.
-
- These analog acoustic delay lines were, in turn replaced in the early 1970s by storage devices based on digital computer technology. The use of digital delay lines requires that the output of the MTI receiver phase-detector be quantized into a sequence of digital words.
 - One of the advantages of a time-domain delay-line canceller as compared to the more conventional frequency-domain filter is that a single network operates at all ranges and does not require a separate filter for each range resolution cell.
 - Frequency-domain Doppler filter banks are of interest in some forms of MTI and Pulse-Doppler radar.

Filter Characteristics of the Delay Line Canceller

- The delay-line canceler acts as a filter which rejects the d-c component of clutter. Because of its periodic nature, the filter also rejects energy in the vicinity of the pulse repetition frequency and its harmonics.
- The video signal of Eq.(3) received from a particular target at a range R_0 is

$$V_1 = k \sin (2\pi f_d t - \phi_0) \dots\dots\dots 4$$

where θ_0 = phase shift and k = amplitude of video signal.

- The signal from the previous transmission, which is delayed by a time T = pulse repetition interval, is

$$V_2 = k \sin [2\pi f_d(t - T) - \theta_0] \quad \text{----- 5}$$

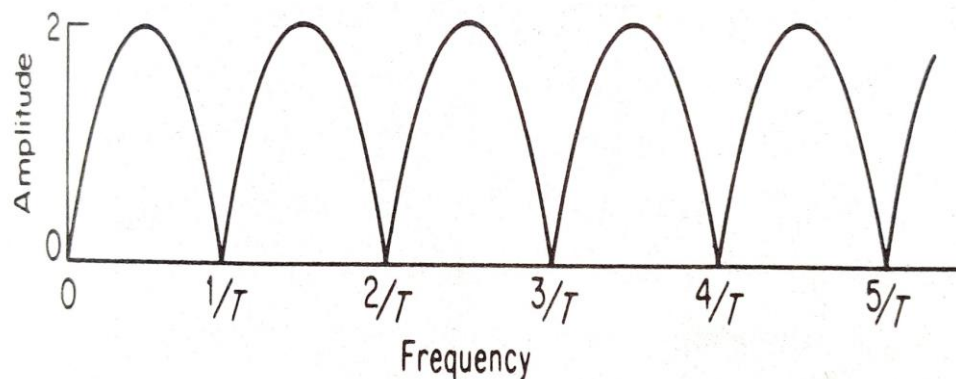
- The output from the subtractor is

$$V = V_1 - V_2 = 2k \sin(\pi f_d T) \cos [2\pi f_d(t - T/2) - \theta_0] \quad \text{----- 6}$$

$$\sin a \pm \sin b = 2 \sin \{1/2 (a \pm b)\} \cos \{1/2 (a \mp b)\}$$

- It is assumed that the gain through the delay-line canceller is unity. The output from the canceller consists of a cosine wave at the Doppler frequency & with an amplitude $2k \sin \pi f_d T$.
- Thus, the amplitude of the canceled video output is a function of the Doppler frequency shift and the pulse-repetition interval, or prf.
- The magnitude of the relative frequency-response of the delay-line canceller [ratio of the amplitude of the output from the delay-line canceller, $2k \sin (\pi f_d T)$ to the amplitude of the normal radar video k] is shown below.

Frequency response of the single delay-line canceller: T = delay time = $1/f_p$



Blind speeds

- The response of the single-delay-line canceller will be zero whenever the argument ($\pi f_d T$) in the amplitude factor of Eq. (6) is 0, π , 2π , ..., etc., or when

$$f_d = \frac{n}{T} = n f_p \quad \text{..... (7)}$$

where $n = 0, 1, 2, \dots$, and f_p = pulse repetition frequency.

- The delay-line canceller not only eliminates the d-c component caused by clutter ($n = 0$), but unfortunately it also rejects any moving target whose Doppler frequency happens to be the same as the prf or a multiple thereof.
- Those relative target velocities which result in zero MTI response are called **blind speeds** and are given by

$$v_n = \frac{n\lambda}{2T} = \frac{n\lambda f_p}{2} \quad n = 1, 2, 3, \dots \quad \text{..... (8)}$$

- where v_n is the n^{th} blind speed. If λ is measured in meters, f_p in Hz, and the relative velocity in knots, the blind speeds are :

$$v_n = \frac{n\lambda f_p}{1.02} \approx n\lambda f_p \quad \text{-----(9)}$$

- The blind speeds are one of the limitations of pulse MTI radar which do not occur with CW radar. They are present in pulse radar because Doppler is measured by discrete samples (pulses) at the prf rather than continuously.
- Based on eq 9 there are four methods for reducing the detrimental effects of blind speeds:
 1. Operate the radar at long wavelengths
 2. Operate with a high pulse repetition frequency
 3. Operate with more than one PRF (Staggered prf's)
 4. Operate with more than one RF frequency
- Unfortunately, there are usually constraints other than blind speeds which determine the wavelength and the pulse repetition frequency. Therefore, blind speeds might not be easy to avoid.
- The possible solution for the blind speed is keep the first blind speed out of the expected range of Doppler frequency

Double cancellation

- The frequency response of a single-delay-line canceller (Fig. 7) does not always have as broad a clutter-rejection null as might be desired in the vicinity of d-c.
- The clutter-rejection notches may be widened by passing the output of the delay-line canceller through a second delay-line canceller as shown in **Fig. (9) below**.
- The output of the two single-delay line cancellers in cascade is the square of that from a single canceller. Thus the frequency response is **($4 \sin^2 \pi f_d T$)**.
- The configuration of **Fig. 9 (a)** is called a double-delay-line canceller, or simply a **double canceller**.
- The relative response of the double canceller compared with that of a single-delay-line canceller is shown in Fig. 10.
- The finite width of the clutter spectrum is also shown (hatched) in this figure so as to illustrate the additional cancellation of clutter offered by the double canceller.
- The two-delay-line configuration of Fig.9 (b) has the same frequency-response characteristic as the double-delay-line canceller.

Figure 9 : (a) Double-delay-line canceller (b) three-pulse canceller

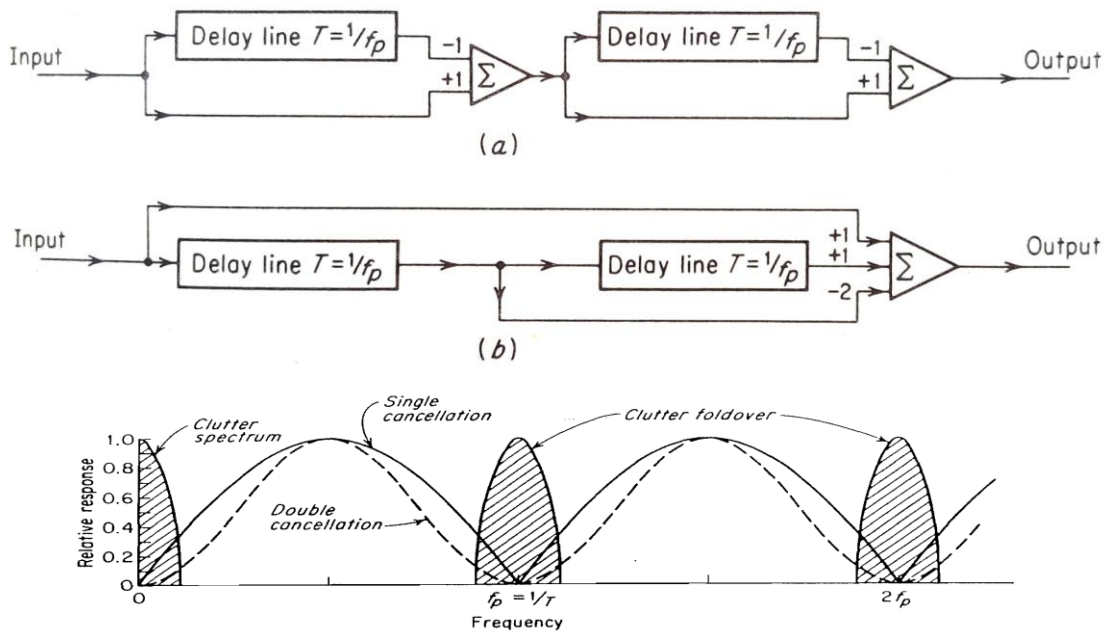


Figure (10): Relative frequency response of the single-delay-line canceller (solid curve) and the double delay-line canceller (dashed curve). Shaded area represents clutter spectrum.

- The operation of the device is as follows. A signal $f(t)$ is inserted into the adder along with the signal from the preceding pulse period, with its amplitude weighted by the factor - 2, plus the signal from the previous two pulse periods.
- The output of the adder is therefore $f(t) - 2f(t + T) + f(t + 2T)$
- which is the same as the output from the double-delay-line canceller

$$f(t) - f(t + T) - f(t + T) + f(t + 2T)$$

- This configuration is commonly called the three-pulse canceller.

Multiple or staggered Pulse Repetition Frequencies

- The use of more than one pulse repetition frequency offers additional flexibility in the design of MTI Doppler filters.
- It not only reduces the effect of the blind speeds , but it also allows a sharper low-frequency cutoff in the frequency response.
- The blind speeds of two independent radars operating at the same frequency will be different if their pulse repetition frequencies are different.
- Therefore, if one radar were “blind ”to moving targets, it would be unlikely that the other radar would be” blind” also.
- Instead of using two separate radars, the same result can be obtained with one radar which time-shares its pulse repetition frequency between two or more different values (**multiple PRF's**).
- The pulse repetition frequency might be switched every other scan or every time the antenna is scanned a half beam width, or the period might be alternated on every other pulse. When the switching is pulse to pulse, it is known as a **staggered PRF**.

An example of the composite (average) response of an MTI radar operating with two separate pulse repetition frequencies on a time-shared basis is shown below.

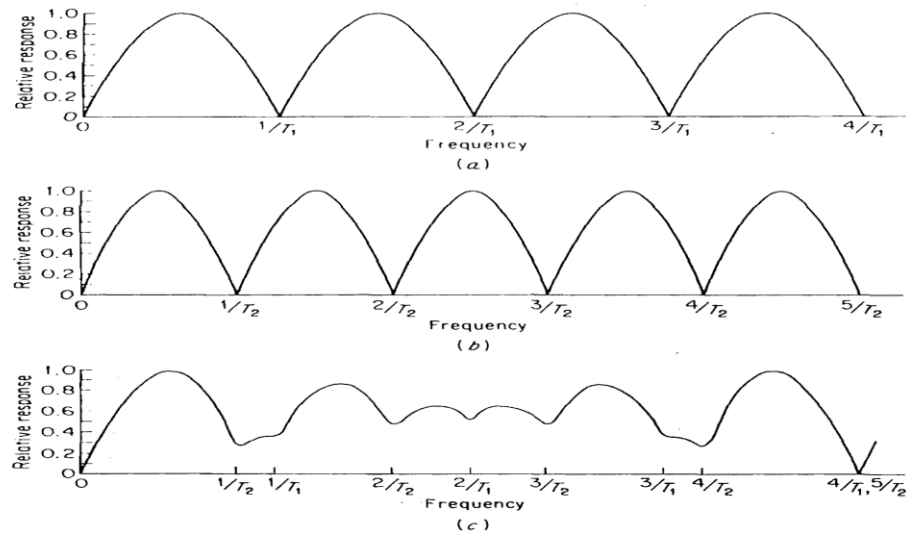
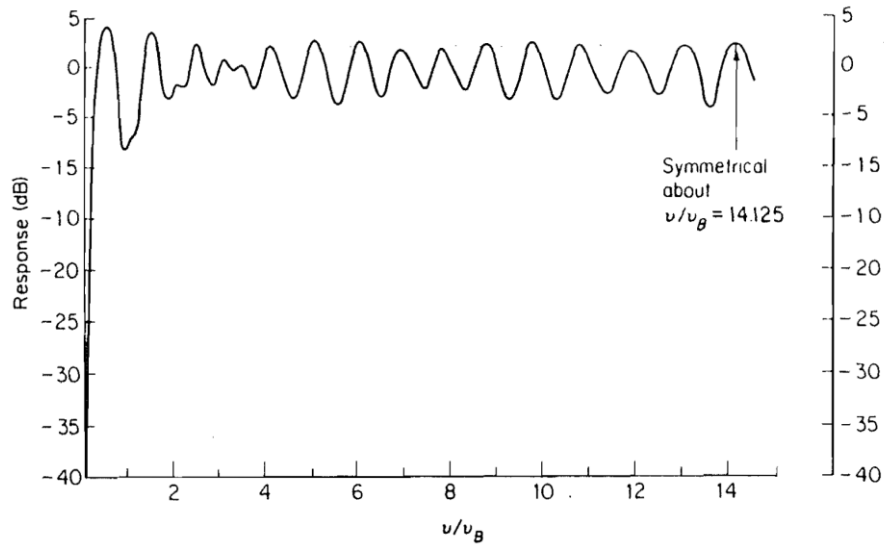


Figure : Frequency-response of a single-delay-line canceller for $f_p = 1/T_1$ (b) same for $f_p = 1/T_2$ (c) Composite response with $T_1/T_2 = 4/5$.

- Zero frequency response occurs only when the blind speeds of both radars coincides.
- The disadvantage is that the region of low sensitivity might appear. (that means may not detect the weak signals)
- As closer the ratio of $T_1:T_2$ is unity, the lower the value of the first blind speed.
- But first null in the vicinity of $f_d = 1/T_1$ becomes deeper.

- Figure below shows the response of a five-pulse stagger (four periods) that might be used with a long-range air traffic control radar.

Figure 12: Frequency response of a five-pulse (four-period) stagger.



- If the periods of the staggered waveforms have the relationship $\mathbf{n}_1/T_1 = \mathbf{n}_2/T_2 = \dots = \mathbf{n}_N/T_N$, where $\mathbf{n}_1, \mathbf{n}_2, \dots, \mathbf{n}_N$ are integers, and if $\mathbf{v}_B$ is equal to the first blind speed of a non-staggered waveform with a constant period equal to the average period $T_{av} = (T_1 + T_2 + \dots + T_N)/N$ then the first blind speed $\mathbf{v}_1$ is given by :

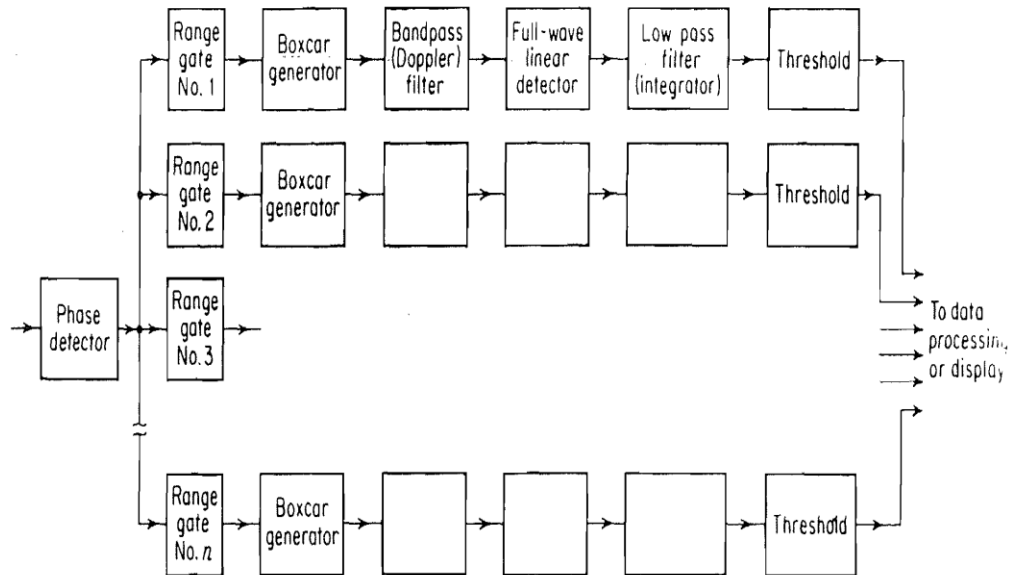
$$\frac{\mathbf{v}_1}{\mathbf{v}_B} = \frac{\mathbf{n}_1 + \mathbf{n}_2 + \dots + \mathbf{n}_N}{N}$$

- A disadvantage of the staggered prf is its inability to cancel second-time-around clutter echoes. Such clutter does not appear at the same range from pulse to pulse and thus produces un canceled residue.
- Second-time-around clutter echoes can be removed by use of a constant prf, providing there is pulse-to-pulse coherence as in the power amplifier form of MTI

Range-gated Doppler filters

- The delay-line canceller, which can be considered as a time-domain filter, is widely used in MTI radar to separate moving targets from stationary clutter.
 - It is also possible to employ the more common frequency-domain band pass filters of conventional design in MTI radar to separate the Doppler-frequency-shifted targets.
 - However the filter configuration would be more complex, than the single, narrow-band pass filter.
 - The narrowband filter "smears" the input pulse since the impulse response is approximately the reciprocal of the filter bandwidth.
 - This smearing destroys the range resolution.
 - If more than one target is present they cannot be resolved.
 - Even if only one target is present, the noise from the other range cells that do not contain the target will interfere with the desired target signal.
 - The result is a reduction in sensitivity due to a collapsing loss. (This Loss Results When Radar Integrates Additional Noise Samples Along with Wanted (S/N) Pulses)
-
- The loss of the range information and the collapsing loss may be eliminated by first quantizing the range (time) into small intervals. This process is called **range gating**.
 - The width of the range gates depends upon the range accuracy desired and the complexity which can be tolerated, but they are usually of the order of the pulse width.
 - Range resolution is established by gating.
 - Once the radar return is quantized into range intervals, the output from each gate may be applied to a narrowband filter.
 - A collapsing loss does not take place since noise from the other range intervals is excluded.

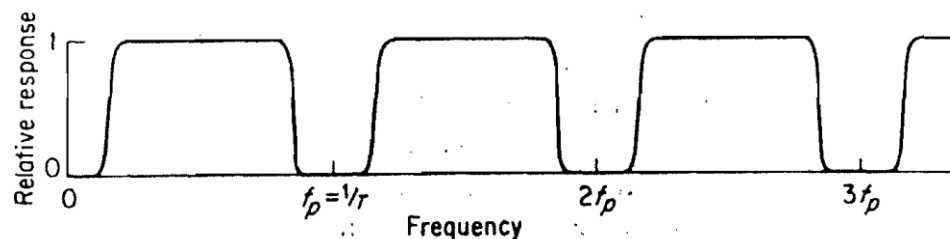
Figure 13: Block diagram of MTI radar using range gates and filters



- A block diagram of the video of an MTI radar with multiple range gates followed by clutter-rejection filters is shown below.
- The output of the phase detector is sampled sequentially by the range gates. Each range gate opens in sequence just long enough to sample the voltage of the video waveform corresponding to a different range interval in space.
- The range gate acts as a switch or a gate which opens and closes at the proper time
- The range gates are activated once each pulse-repetition interval.
- The output for a stationary target is a series of pulses of constant amplitude.
- An echo from a moving target produces a series of pulses which vary in amplitude according to the Doppler frequency.
- The output of the range gates is stretched in a circuit called the **boxcar generator**, or **sample-and-hold** circuit, whose purpose is to aid in the filtering and detection process by emphasizing the fundamental of the modulation frequency and eliminating harmonics of the pulse repetition frequency.

- The clutter rejection filter is a band pass filter whose bandwidth depends upon the extent of the expected clutter spectrum.
- Following the Doppler filter is a full-wave linear detector and an integrator (a low-pass filter). The purpose of the detector is to convert the bipolar video to unipolar video.
- **The** output of the integrator is applied to a threshold-detection circuit. Only those signals which cross the threshold are reported as targets.
- Following the threshold detector, the outputs from each of the range channels must be properly combined for display on the PPI or A-scope or for any other appropriate indicating or data-processing device.
- The CRT display from this type of MTI radar appears "cleaner" than the display from a normal MTI radar, not only because of better clutter rejection, but also because the threshold device eliminates many of the unwanted false alarms due to noise.

Figure Frequency-response characteristic of an MTI using range gates and filters.



The band pass filter can be designed with a variable low-frequency cutoff that can be selected to conform to the prevailing clutter conditions.

A variable lower cutoff might be advantageous when the width of the clutter spectrum changes with time as when the radar receives unwanted echoes from birds.

A relatively wide notch at zero frequency is needed to remove moving birds.

If the notch were set wide enough to remove the birds, it might be wider than necessary for ordinary clutter and desired targets might be removed.

Since the appearance of birds varies with the time of day and the season, it is important that the width of the notch be controlled according to the local conditions.

- MTI radar using range gates and filters is usually more complex than an MTI with a single-delay-line canceller.
- The additional complexity is justified in those applications where good MTI performance and the flexibility of the range gates and filter MTI are desired.
- The better MTI performance results from the better match between the clutter filter characteristic and the clutter spectrum.

Limitations to MTI Performance

The improvement in signal-to-clutter ratio of an MTI is affected by factors other than the design of the Doppler signal processor such as:

- Instabilities of the transmitter and receiver
 - physical motions of the clutter
 - Finite time on target (or scanning modulation)
 - And limiting in the receiver
- Before discussing these limitations, we shall study the *related definitions*

Definitions related to MTI Performance

MTI improvement factor: The signal-to-clutter ratio at the output of the MTI system divided by the signal-to-clutter ratio at the input, averaged uniformly over all target radial velocities of interest.

With respect to Doppler frequency, the Improvement factor can be expressed as:

$$I_f = (S/C)_{out} * (S/C)_{in} = \frac{C_{in}}{C_{out}} G_{av}$$

C_{in} = strength of clutter at clutter filter input

C_{out} = strength of clutter at clutter filter output

G_{av} = average filter gain for moving targets

Subclutter Visibility (SCV):

It describes the radar's ability to detect non-stationary targets embedded in a strong [clutter](#) background with a given signal-to-clutter ratio (SCR), for some specified [probabilities of detection](#) and [false alarm](#). It is often used as a measure of the effectiveness of moving-target indicator radar, equal to the ratio of the signal from a fixed target that can be canceled to the signal from a just visible moving target.

The subclutter visibility is expressed as the ratio of the improvement factor to the minimum [MTI](#) output given signal-to-clutter ratio (SCR) required for proper detection for a given probability of detection.

$$SCV = \frac{I_f}{(SCR)_{out}}$$

SCV = Sub-Clutter Visibility SCR = Signal-to-Clutter Ratio

Interclutter visibility(ICV)

If a radar system can resolve the areas of strong and weak clutter within its field of view, then the phrase Interclutter visibility (ICV) describes the ability to recognize the moving target which occur in clear resolution cells between strong clutter patches.

Clutter visibility factor: The signal-to-clutter ratio, after cancellation or Doppler filtering that provides stated probabilities of detection and false alarm.

Clutter attenuation: The ratio of clutter power at the canceller input to the clutter residue at the output, normalized to the attenuation of a single pulse passing through the unprocessed channel of the canceller. (The **clutter residue** is the clutter power remaining at the output of a MTI system.)

Cancellation ratio: The ratio of canceller voltage amplification for the fixed-target echoes received with a fixed antenna, to the gain for a single pulse passing through the unprocessed channel of the canceller.

Equipment instabilities :

- Pulse-to-pulse changes in the amplitude, frequency, or phase of the transmitter signal, changes in the stalo or coho oscillators in the receiver, jitter in the timing of the pulse transmission, variations in the time delay through the delay lines, and changes in the pulse width can cause the apparent frequency spectrum from perfectly stationary clutter to broaden and thereby lower the improvement factor of an MTI radar.
- The stability of the equipment in MTI radar must be considerably better than that of an ordinary radar. It can limit the performance of MTI radar if sufficient care is not taken in design, construction, and maintenance.

Internal fluctuation of clutter :

- Although clutter targets such as buildings, water towers, bare hills. or mountains produce echo signals that are constant in both phase and amplitude as a function of time, there are many types of clutter that cannot be considered as absolutely stationary.
- Echoes from trees, vegetation, sea, rain fluctuate with time, and these fluctuations can limit the performance of MTI radar.

Antenna scanning modulation

- As the antenna scans by a target, it observes the target for a finite time equal to : $t_o = n_B / f_p = \theta_B / \theta_s$ where n_B = number of hits received, f_p = pulse repetition frequency, θ_B, θ_s = antenna beamwidth and antenna scanning rate.
- The received pulse train of finite duration t_o has a frequency spectrum (which can be found by taking the Fourier transform of the waveform) whose width is proportional to $1/t_o$.
- Therefore, even if the clutter were perfectly stationary, there will still be a finite width to the clutter spectrum because of the finite time on target.
- If the clutter spectrum is too wide because the observation time is too short, it will affect the improvement factor. This limitation has sometimes been called scanning fluctuations or scanning modulation.

Limiting in MTI Radar:

- A limiter is usually employed in the IF amplifier just before the MTI processor to prevent the residue from large clutter echoes from saturating the display. Ideally a MTI radar should reduce the clutter to a level comparable to receiver noise.
- However, when the **MTI improvement factor** is not great enough to reduce the clutter sufficiently, the clutter residue will appear on the display and prevent the detection of aircraft targets whose cross sections are larger than the clutter residue. This condition may be prevented by setting the limit level L , relative to the noise N , equal to the MTI improvement factor I ; or $L/N = I$.
- If the limit level relative to noise is set higher than the improvement factor. Clutter residue obscures part of the display. If it is set too low, there may be a "black hole" effect on the display. The limiter provides a constant false alarm rate (CFAR) and is essential to usable MTI Performance.
- Unfortunately, nonlinear devices such as limiters have side-effects that can degrade performance. Limiters cause the spectrum of strong clutter to spread into the canceller pass- band, and result in the generation of additional residue that can significantly degrade MTI performance as compared with a perfect linear system.

Benefits or advantages of MTI Radar

- MTI radar can distinguish between moving target and stationary target.
- It uses low PRF (Pulse Repetition Frequency) to avoid range ambiguities.
- MTI principle is used in air surveillance radar which operates in presence of clutter.
- It is simpler compare to pulse doppler radar.
- Antenna bandwidth is high.
- It is economical.
- It does not require waveforms with multiple PRF.
- It is preferred at UHF frequencies.

Drawbacks or disadvantages of MTI Radar

- Blind speed does not get detected by pulse MTI radar. Blind speed is defined as magnitude of radial component of velocity of target when moving target appears as stationary target.
- They can have doppler ambiguities.

MTI VS PULSE DOPPLER RADAR

- A Pulse radar that extracts the Doppler frequency shift for the purpose of detecting moving targets in the presence of clutter is either a **MTI Radar** or a **Pulse Doppler Radar**.
- The distinction between them is based on the fact that in a sampled measurement system like a pulse Radar, ambiguities arise in measuring both the Doppler frequency (relative velocity) and the Range (time delay).
- Range ambiguities are avoided with a **low** sampling rate (low pulse repetition frequency), and Doppler frequency ambiguities are avoided with a **high** sampling rate.
- However, in most radar applications the sampling rate, or pulse repetition frequency, cannot be selected to avoid both types of measurement ambiguities.
- Therefore, a compromise must be made and the nature of the compromise generally determines whether the radar is called an **MTI** or a **Pulse Doppler Radar**.

- MTI usually refers to a Radar in which the pulse repetition frequency is chosen low enough to avoid ambiguities in range (no multiple-time-around echoes) but with the consequence that the frequency measurement is ambiguous and results in blind speeds.
- The pulse Doppler radar, on the other hand, has a high pulse repetition frequency that avoids blind speeds, but it experiences ambiguities in range.
- The pulse Doppler radar is more likely to use range-gated Doppler filter-banks than delay-line cancellers. Also, a power amplifier such as a klystron is more likely to be used than a power oscillator like the magnetron.
- A pulse Doppler radar operates at a higher duty cycle than does an MTI.
- Although it is difficult to generalize, the MTI radar seems to be the more widely used of the two, but pulse Doppler radar is usually more capable of reducing clutter.

TRACKING RADAR

Tracking with Radar

Sequential Lobing

Conical Scan

Monopulse Tracking Radar

Amplitude Comparison Monopulse (One and two coordinate)

Phase Comparison Monopulse

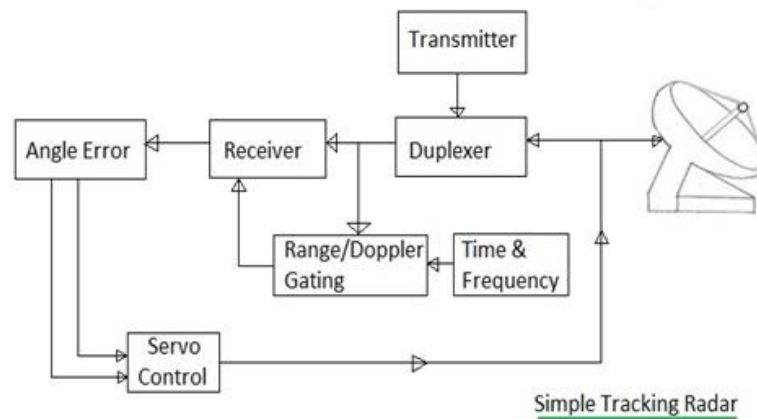
Tracking in Range

Acquisition and Scanning Patterns

Comparison of Trackers

Tracking With Radar

- Tracking is the process of continuously maintaining the antenna beam on the target and also the echo signal within the range gate.
- The radar which detects target and determines location as well as predict its trajectory path as well as its future coordinates is known as tracking radar.
- Based on the measured coordinates error signal will be generated.
- Antenna should be moved based on error signal to maintain the target within the beam.
- Use pencil beam
- The figure below mentions block diagram of simple **tracking radar**.
- As shown tracking operation in the radar depends upon angular information. very narrow antenna beam is used here which will track one target object at one time. This can be performed using range gating and doppler filtering module.
- Range tracking is carried out using timing control unit. Doppler tracking is carried out using Doppler gating unit.



- The angle error signal is provided as input for servo motor based control system. This servo system will steer the antenna as per error input and hence will track the target.
- The various methods for generating the error signal are classified as sequential lobing, conical scan, and simultaneous lobing or Monopulse.
- The data available from a tracking radar may be presented on a cathode-ray-tube (CRT) display

Tracking Radar Types

Following are the types of tracking radar:

- STT Radar (Single Target Tracking Radar)
- ADT Radar (Automatic Detection and Tracking Radar)
- TWS Radar (Track While Scan Radar)
- Phased Array Tracking Radar
- Monopulse Tracking Radar

STT Radar (Single Target Tracking Radar)

- Tracks a single target at fast data
- High Data rate – 10 obs/sec.
- Employs a closed loop servo system to keep the error signal small.
- Application – tracking of aircraft/missile targets

ADT Radar (Automatic Detection and Tracking Radar)

- This Tracking is preferred in air surveillance tracking radar.
- Lower data rate than STT.
- Can track hundreds/ a few thousand targets simultaneously.
- Tracking is open loop i.e antenna position is not controlled by data processing.

TWS Radar (Track While Scan Radar)

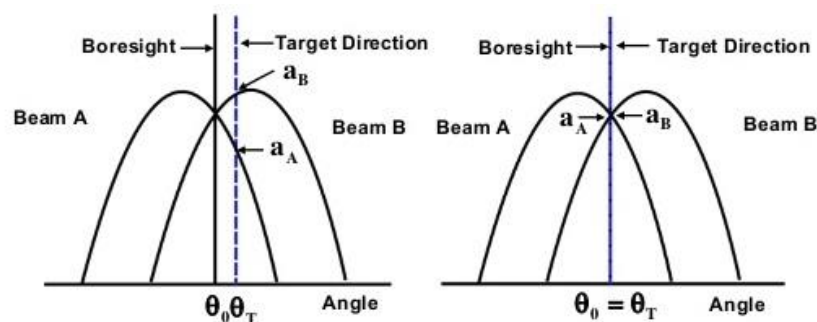
- This rapidly scans a limited angular sector to maintain tracks with a moderate data rate on more than one target within the coverage of antenna (another name for ADT).
- Scans a limited angular sector to maintain tracks – **simultaneous track & search**
- Data rate : moderate
- Can track a number of targets.
- Equivalent of track while scan is **ADT**
- TWS systems are used for air-defense radars, air craft landing radars and in air borne intercept radars to track multiple objects

Phased Array Tracking Radar

- A large number of targets can be held in track
- This is done on time sharing basis
- Beam is electronically switched from one angular position to another in a few microseconds.
- It combines the rapid update rate of a single target tracker with the ability of ADT to hold many targets in track
- High data rate (like in STT)
- The cost is very very high.
- These radars are used in air defense weapon systems.

Angle Tracking

- When a target is approaching, the antenna is to be moved continuously to track the target
- To determine the direction in which the antenna beam needs to be moved, a measurement has to be made at two different beam positions.
- Below figure shows the basic principle of continuous angle tracking

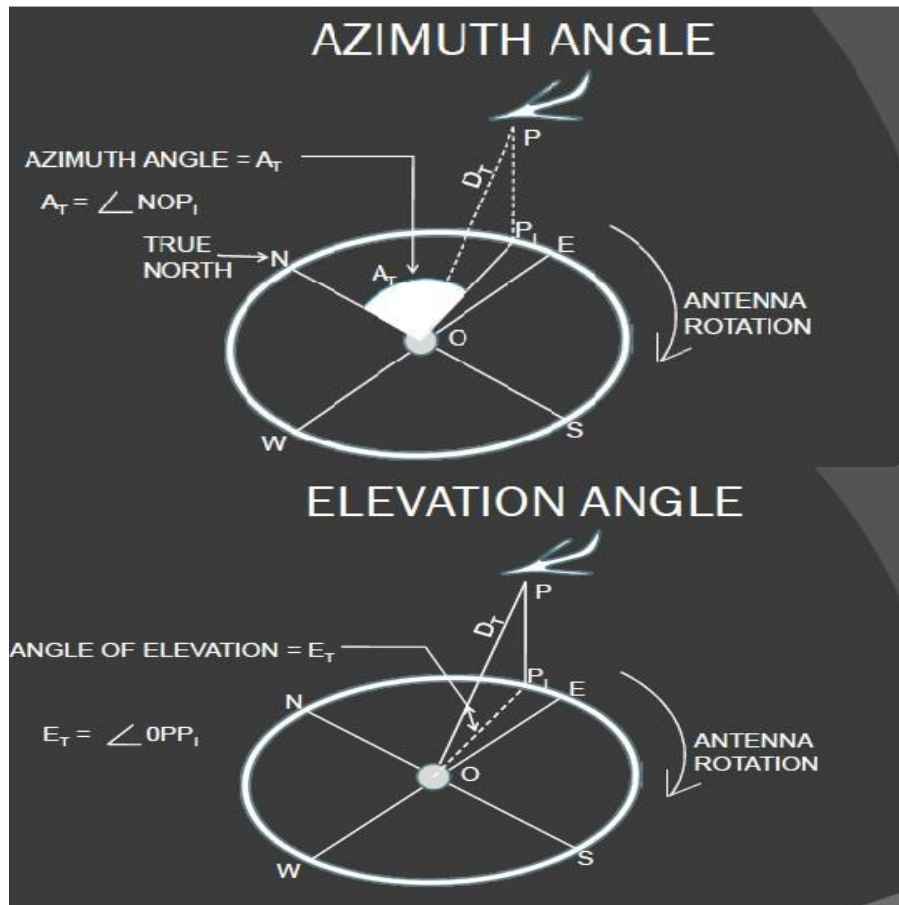


- Two overlapping antenna patterns that crossover at the boresight direction θ_0
- A target is located in this example to the right of the boresight at an angle θ_T
- The amplitude a_B of the target echo in beam B is larger than the amplitude a_A in the beam A
- Which indicates that the two beams should moved to the right to bring the target to the boresight position.
- If you want to track the target the boresight is always maintained in the direction of the target.

Methods to extract error signal may be classified as

- Sequential lobing
 - Conical scan
 - Simultaneous lobing or monopulse
- $\left. \begin{array}{l} \text{Sequential lobing} \\ \text{Conical scan} \end{array} \right\} \begin{array}{l} 1. \text{ Time shares a single beam} \\ 2. \text{ Antenna beam is switched between two positions} \end{array}$
- $\rightarrow \begin{array}{l} 1. \text{ More than one simultaneous beam is used for tracking} \\ 2. \text{ Usually 4 simultaneous beams used for 2-dimensional tracking} \end{array}$

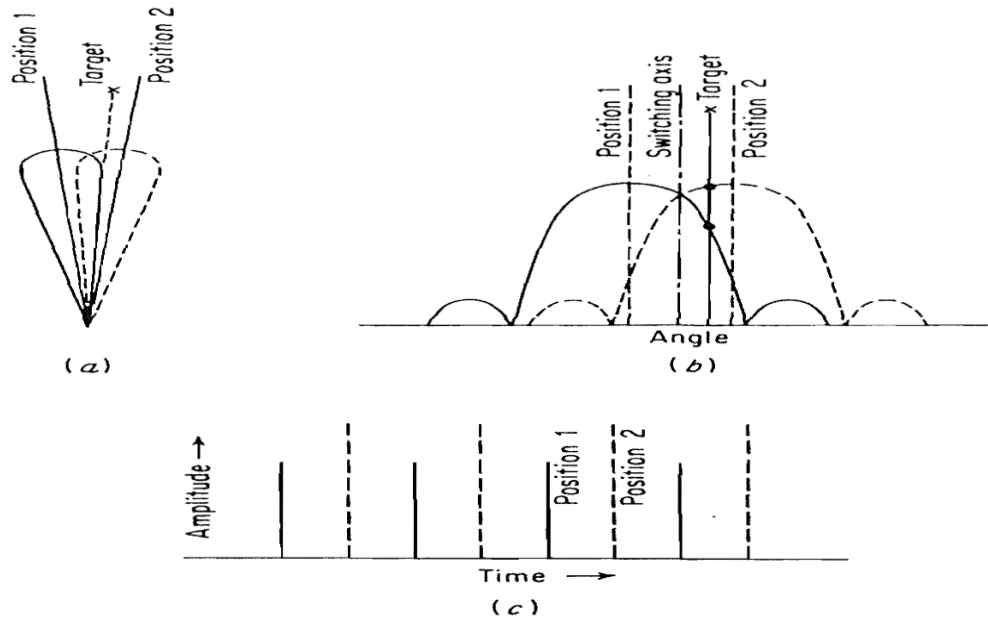
Single beam on time sharing basis.	Multiple beam.
Sequential lobing Radar and Conical scan Radar	Simultaneous lobing or monopulse Radar]
Simpler	Complex
One antenna	Multiple antennas
Less equipment	More equipments
Not accurate	Accurate
No of pulses are required to extract the error signal	Single pulse is used to determine the angular error.



Sequential Lobing

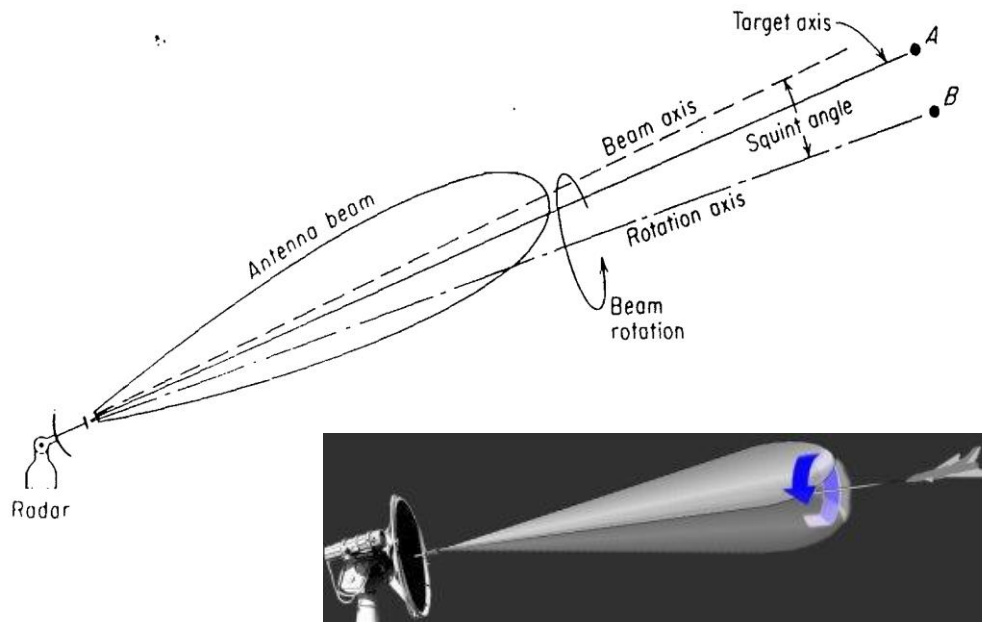
- The antenna pattern commonly employed with tracking radars is the symmetrical pencil beam in which the, elevation and azimuth beam widths are approximately equal.
- Actually the difference between the target position and the reference direction is the angular error.
- The tracking radar attempts to position the antenna to make the angular error zero. When the angular error is zero, the target is located along the reference direction.
- One method of obtaining the direction and the magnitude of the angular error in one coordinate is switching the single antenna beam between two squinted angular positions. This is called as lobe switching, sequential switching or sequential lobing.
- The error signal is obtained from a target not on the switching axis.
- The direction in which to move the beam to bring the target on boresight is found by observing which beam position has the larger signal.

Fig 1-a is a polar representation of the antenna beam (minus the side lobes) in the two switched positions. A plot in rectangular coordinates is shown in **Fig.1-b**, and the error signal obtained from a target not on the switching axis (reference direction) is shown in **Fig.1-c**.



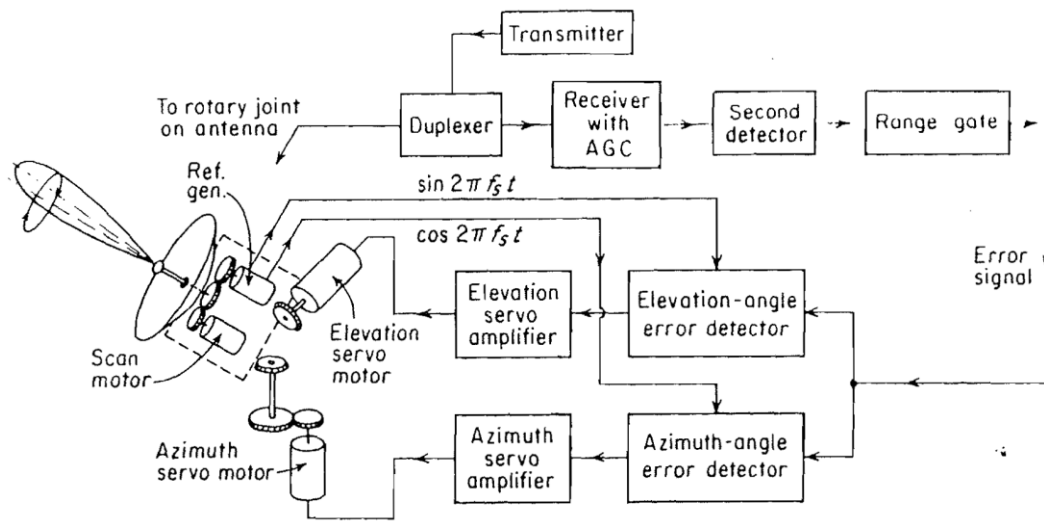
- When the echo signals in the two beam positions are equal, the target is on axis and its direction is that of the switching axis.
- If orthogonal angle information is needed, two more switching positions are needed.
- So, two dimensional sequentially lobing radar might consist of four feed horns illuminating a single reflector antenna.
- An improvement over this can be a single squinted feed which could be rotated continuously to obtain angle measurements in two coordinates. This results in conical scan.
- One of the limitations of a simple unswitched non-scanning pencil-beam antenna is that the angle accuracy can be no better than the size of the antenna beam width.
- An important feature of sequential lobing (as well as the other tracking techniques to be discussed) is that the target-position accuracy can be far better than that given by the antenna beam width.

Conical Scan



- The angle between the axis of rotation and the axis of the antenna beam is called the Squint Angle.
- Consider a target at position A.
- The echo signal amplitude will be modulated at a frequency equal to the rotation frequency of the beam.
- The amplitude of the echo signal modulation will depend upon the shape of the antenna pattern, the squint angle and the angle between the target line of sight & the rotation axis. This amplitude of the echo signal will be modulated at a frequency equal to the beam rotation frequency (conical Scan frequency).
- The phase of the modulation depends on the angle between the target and the rotation axis.
- The conical scan modulation is extracted from the echo signal and applied to a servo-control system which continually positions the antenna rotation axis in the direction of the target. [Note that two servos are required because the tracking is required in two-dimensions.
- When the antenna is on target, as in B of **Fig. 2**, the line of sight to the target and the rotation axis coincide, and the conical-scan modulation is zero.
- Two servo motors are required, one for azimuth and the other for elevation

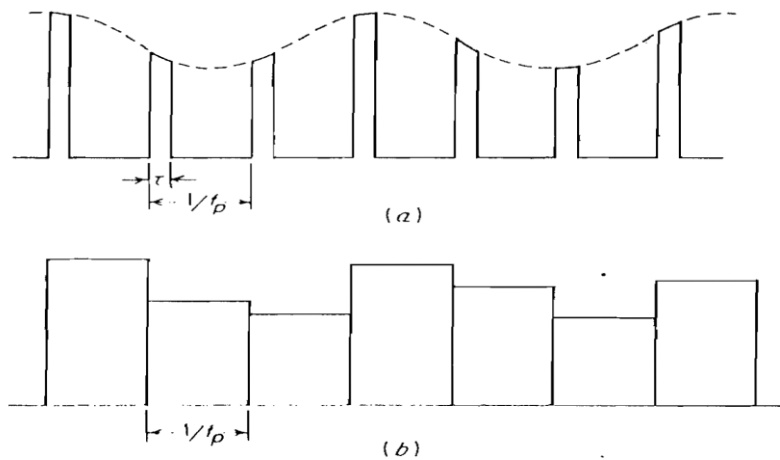
Block diagram of conical scan tracking radar



- The antenna is mounted so that it can be positioned in both azimuth and elevation by separate m
- Redirection of beam i) Rotating feed ii) Nutating feed.
- When the feed is designed to maintain the plane of polarization as it rotates about the axis, it is called 'nutating feed'.
- A rotating feed is one which causes the plane of polarization to rotate.
- The nutating feed is preferred over the rotating feed since a rotating polarization can cause the amplitude of the target echo signal to change with time even for a stationary target on axis.
- A change in amplitude caused by a modulated echo signal can result in degraded angle tracking accuracy.
- The nutating feed is more complicated than the rotating feed.
- A typical conical scan rotation speed might be in the vicinity of 30 rev/sec.
- The same motor that provides the conical-scan rotation of the antenna beam also drives a two-phase reference generator with two sinusoidal outputs 90° apart in phase.

- These two outputs serve as a reference to extract the elevation and azimuth errors.
- The received echo signal is fed to the receiver from the antenna via two rotary joints (not shown in the block diagram). One rotary joint permits motion in azimuth; the other, in elevation.
- The receiver is conventional super heterodyne except for features related to the conical scan tracking.
- The error signal is extracted in the video after the second detector.
- Range gating eliminates noise and excludes other targets.
- The error signal from the range gate is compared with both the elevation and azimuth reference signals in the angle error detectors.
- The angle error outputs are amplified and used to drive the antenna elevation and azimuth servo motors.
- The video signal is a pulse train modulated by the conical scan frequency.
- It is usually convenient to stretch the pulses before low pass filtering so as to increase the energy at the conical scan frequency to perform A/D conversion.
- This pulse stretching is accomplished by a sample-and hold circuit which also known as boxcar generator.

Fig 4: (a) Pulse train with conical scan modulation (b) same pulse train after passing through boxcar generator.(stretching by a sample and hold circuit)



- PRF must be sufficiently large compared to conical scan frequency for proper filtering and avoiding inaccuracy of the angle measurement.
- The PRF must be atleast four times of conical scan frequency but normally 10 times.

Automatic Gain Control (AGC) :

- The echo-signal amplitude at the tracking-radar receiver will not be constant but will vary with time. The three major causes of variation in amplitude are:
 - The inverse-fourth-power relationship between the echo signal and range
 - The conical scan modulation (angle-error signal) and
 - Amplitude fluctuations in the target cross Section.
- The function of the automatic gain control (AGC) is to maintain the d-c level of the receiver output constant and to smooth or eliminate as much of the noise like amplitude fluctuations as possible without disturbing the extraction of the desired error signal at the conical-scan frequency.
- AGC is also important for avoiding saturation by large signals which could cause the loss of the scanning modulation and the accompanying error signal.

An example of the AGC portion of a tracking-radar receiver is shown in Fig

- A portion of the video-amplifier output is passed through a low-pass or smoothing filter and fed back to control the gain of the IF amplifier.
- The larger the video output, the larger will be the feedback signal and the greater will be the gain reduction.

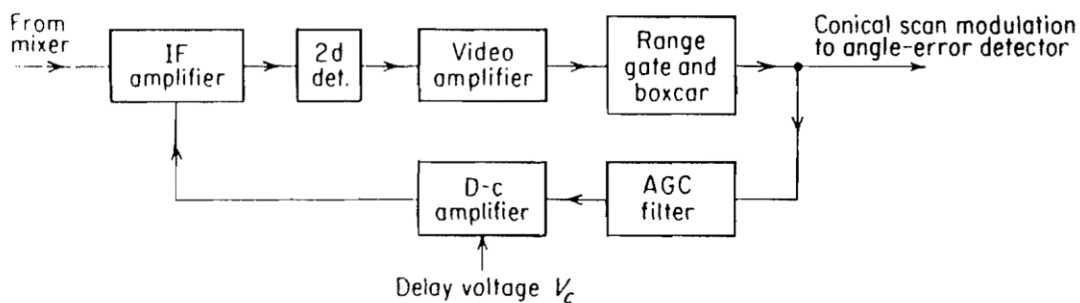


Figure: Block diagram of the AGC portion of a tracking-radar receiver

- The filter in the AGC loop should pass all frequencies from direct current to just below the conical-scan-modulation frequency.
- The loop gain of the AGC filter measured at the conical-scan frequency should be low so that the error signal will not be affected by AGC action.
- The phase shift of this filter must be small if its phase characteristic is not to influence the error signal.
- A phase change of the error signal is equivalent to a rotation of the reference axes and introduces cross coupling, or "cross talk," between the elevation and azimuth angle-tracking loops.

- Cross talk affects the stability of the tracking and might result in an unwanted nutating motion of the antenna.
- In conventional tracking radar applications, the phase change introduced by the feedback-loop filter should be less than 10° and in some applications, it should be as little as 2° .
- For this reason, a filter with a sharp attenuation characteristic in the vicinity of the conical-scan frequency might not be desirable because of the relatively large amount of phase shift which it would introduce.

Other considerations:

- In both the sequential-lobing and conical-scan techniques, the measurement of the angle error in two orthogonal coordinates (azimuth and elevation) requires that a minimum of three pulses be processed.
- In practice, however, the minimum number of pulses in sequential lobing is usually four-one per quadrant. Although a conical scan radar can also be operated with only four pulses per revolution, it is more usual to have ten or more per revolution. This allows the modulation due to the angle error to be more that of a continuous sine wave.
- Thus, the **PRF** is usually at least an order of magnitude greater than the conical-scan frequency.
- The scan frequency also must be at least an order of magnitude greater than the tracking bandwidth.
- A conical-scan-on-receive-only (COSRO) tracking radar radiates a non-scanning transmit beam, but receives with a conical scanning beam to extract the angle error. The analogous operation with sequential lobing is called lobe-on-receive-only (LORO).

DISADVANTAGES

Sequential lobing

- Angle accuracy can be no better than the size of the antenna beamwidth.
- Variation in echo strength on a pulse-by-pulse basis changes the signal level thereby reducing tracking accuracy
- The antenna gain is less than the peak gain in beam axis direction, reducing maximum range that can be measured

Conical scan

- The antenna scan rate is limited by the scanning mechanism (mechanical or electronic)
- Sensitive to target modulation
- Mechanical vibration and wear and tear due to rotating feed
- It creates confusion by rapid changes in signal strength

MONOPULSE TRACKING RADAR

- Pulse-to-pulse amplitude fluctuations of the echo signal have no effect on tracking accuracy if the angular measurement is made on the basis of one pulse rather than many.
- There are several methods by which angle-error information might be obtained with only a single pulse.
- More than one antenna beam is used simultaneously in these methods, in contrast to the conical-scan or lobe-switching tracker, which utilizes one antenna beam on a time-shared basis.
- The angle of arrival of the echo signal may be determined in a single-pulse system by measuring the relative phase or the relative amplitude of the echo pulse received in each beam.
- The names **simultaneous lobing** and **monopulse** are used to describe those tracking techniques which derive angle-error information on the basis of a single pulse.
- Most popular monopulse is – Amplitude Comparison Monopulse

Amplitude Comparison Monopulse

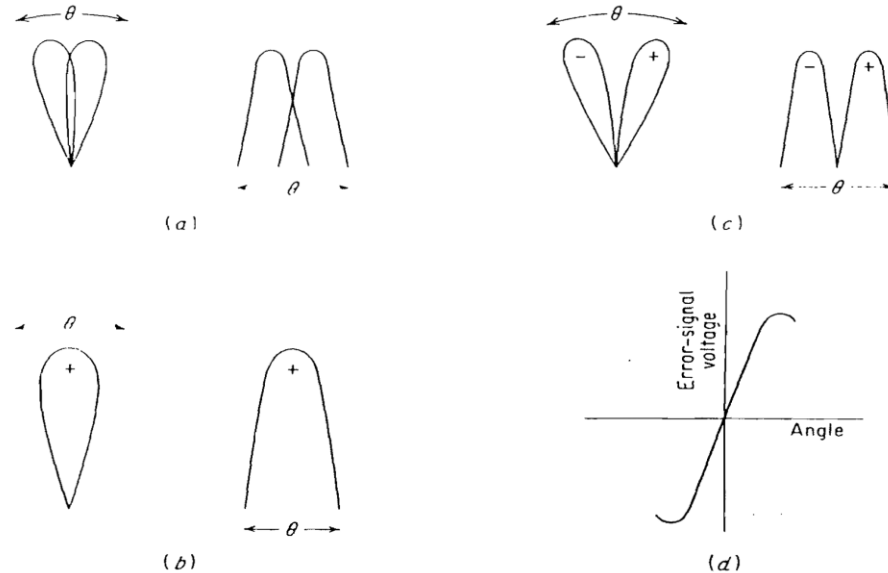


Figure: Monopulse antenna patterns and error signal. Left-hand diagrams in (a-c) are in polar coordinates. Right-hand diagrams are in rectangular coordinates. (a) Overlapping antenna patterns (b) sum pattern (c) difference pattern (d) product (error) signal.

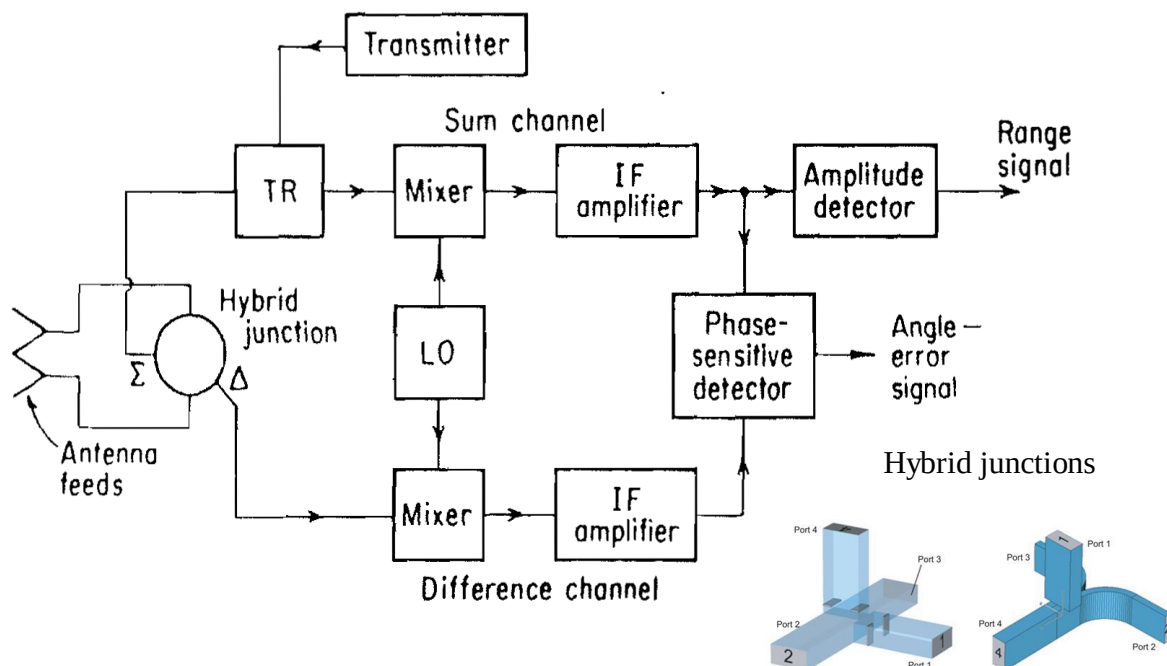
- In this technique, the RF signals received from two offset antenna beams are combined so that both the sum and the difference signals are obtained simultaneously.
- The sum and difference signals are multiplied in a **phase-sensitive detector** to obtain both the magnitude and the direction of the error signal.
- All the information necessary to determine the angular error is obtained on the basis of a single pulse; hence the name **monopulse**.
- The amplitude-comparison monopulse employs two overlapping antenna patterns to obtain the angular error in one coordinate.
- The sum of the two antenna patterns of **Fig (a)** is shown in **Fig (b)**, and the difference in **Fig(c)**.
- The sum pattern is used for transmission, while both the sum pattern and the difference pattern are used on reception.
- The signal received with the difference pattern provides the magnitude of the angle error.
- The sum signal provides the range measurement and is also used as a reference to extract the sign of the error signal.
- Signals received from the sum and the difference patterns are amplified separately and combined in a phase-sensitive detector to produce the error-signal characteristic shown in **Fig (d)**.

Phase-sensitive detector

- The system contains a phase sensitive detector that compares two signals of the same frequency.
- It is a nonlinear device
- The output indicates the direction of the angle error relative to the boresight.
- Though phase comparison is done, the magnitude of the angle error signal is determined by comparison of amplitude signals.

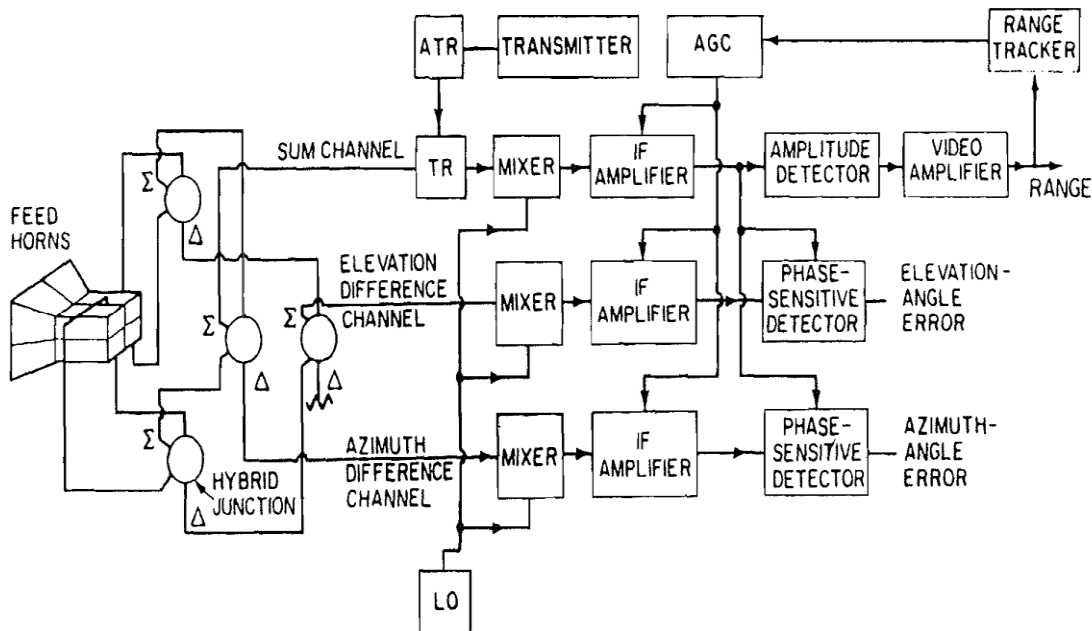
Amplitude-comparison monopulse (One angular coordinate)

- A block diagram of the amplitude-comparison-monopulse tracking radar for a single angular coordinate is shown in below Fig.



- The two adjacent antenna feeds are connected to the two arms of a **hybrid junction** such as a "**magic T**," a "**rat race**," or a "**short-slot coupler**".
 - The sum and difference signals appear at the two other arms of the **hybrid**. On reception, the outputs of the sum arm and the difference arm are each heterodyned to an intermediate frequency and amplified as, in any super heterodyne receiver.
 - The transmitter is connected to the sum arm. Range information is also extracted from the sum channel.
 - A duplexer is included in the sum arm for the protection of the receiver.
 - The output of the phase-sensitive detector is an error signal whose magnitude is proportional to the angular error and whose sign is proportional to the direction.
 - The output of the monopulse radar is used to perform automatic tracking.
 - The angular error signal actuates a servo-control system to position the antenna, and the range output from the sum **channel feeds** into an automatic-range-tracking unit.
-
- The sign of the difference signal (and the direction of the angular error) is determined by comparing the phase of the difference signal with the phase of the sum signal.
 - If the sum signal in the IF portion of the receiver were $A_s \cos(\omega_{IF}t)$ the difference signal would be either $A_d \cos(\omega_{IF}t)$ or $-A_d \cos(\omega_{IF}t)$ ($A_s > 0$, $A_d > 0$), depending on which side of center is the target.
 - Since $-A_d \cos(\omega_{IF}t) = A_d \cos(\omega_{IF}(t+\pi))$, the sign of the difference signal may be measured by determining whether the difference signal is in phase with the sum or **180°** out of phase.
 - The purpose of the phase-sensitive detector is **only** to conveniently furnish the **sign** of the error signal.

Amplitude-comparison monopulse (Two angular Coordinate)



- A block diagram of a monopulse radar with provision for extracting error signals in both elevation and azimuth is shown in above **Fig**
- The cluster of four feeds generates four partial overlapping antenna beams. The feeds might be used with a parabolic reflector, Cassegrain antenna, or a lens.
- All four feeds generate the sum pattern. The difference pattern in one plane is formed by taking the sum of two adjacent feeds and subtracting this from the sum of the other two adjacent feeds.
- The difference pattern in the orthogonal plane is obtained by adding the differences of the orthogonal adjacent pairs.
- A total of four hybrid junctions generate the sum channel, the azimuth difference channel, and the elevation difference channel.
- Three separate mixers and IF amplifiers are shown, one for each channel.
- All three mixers operate from a single local oscillator in order to maintain the phase relationships between the three channels.
- Two phase-sensitive detectors extract the angle-error information, one for azimuth, the other for elevation.
- Range information is extracted from the output of the sum channel after amplitude detection.

- An alternative approach to using three identical amplifiers in the monopulse receiver is to use only one IF channel which amplifies the sum signal and the two difference signals on a time-shared basis.
- The sum signal is passed through the single IF amplifier followed by the two difference signals delayed in time by a suitable amount.
- Automatic gain control (AGC) is required in order to maintain a stable closed-loop servo system for angle tracking and to insure that angle error signal is not affected by changes in the received signal amplitude.
- The AGC results in a constant angle sensitivity independent of target size and range.
- With AGC the output of the angle-error detector is proportional to the difference signal normalized (divided) by the sum signal. The output of the sum channel is constant.

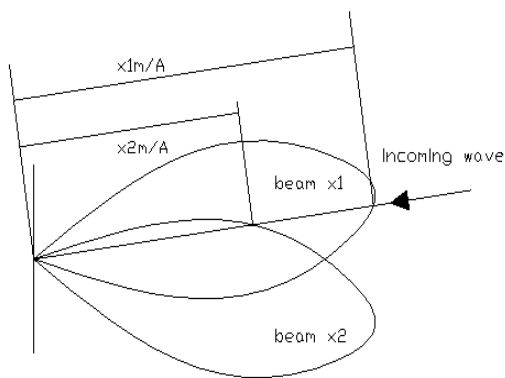
$$\begin{aligned} \text{[SUM PATTERN : } A+B +C +D \\ \text{AZIMUTH DIFF. PATTERN : } (A+B) - (C +D) \\ \text{ELEVATION DIFF. PATTERN : } (B+D) - (A+C)] \end{aligned}$$

Important Requirements of Amplitude-comparison monopulse

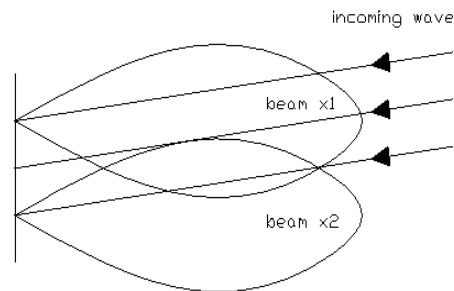
- The monopulse antenna must generate a sum pattern with high efficiency (maximum boresight gain), and a difference pattern with a large value of slope at the crossover of the offset beams.
- *The greater the signal-to-noise ratio and the steeper the slope of the error signal in the vicinity of zero angular error, the more accurate is the measurement of angle.*
- Furthermore, the side lobes of both the sum and the difference patterns must be low.
- The antenna must be capable of the desired bandwidth, and the patterns must have the desired polarization characteristics.

Phase comparison Monopulse

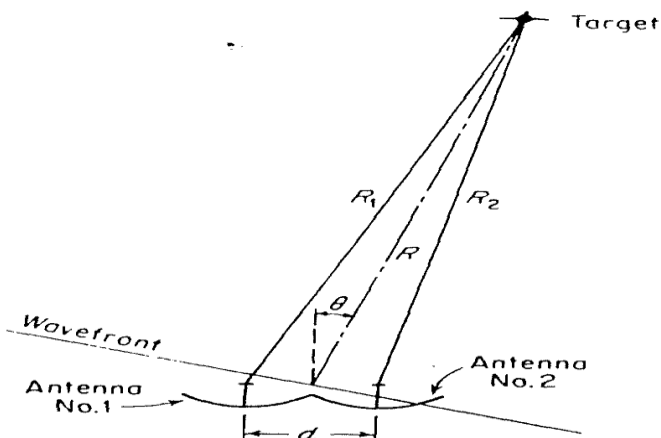
- The tracking techniques discussed thus far in this chapter are based on the comparison of the amplitude of echo signals received from two or more antenna positions.
- The sequential-lobing and conical-scan techniques used a single, time-shared antenna beam while the Monopulse technique used two or more simultaneous beams.
- The difference in amplitudes in the several antenna positions is proportional to the angular error.
- A tracking radar which operates with phase information is similar to an **active interferometer** and is also called **interferometer radar**. It has also been called **simultaneous phase comparison radar** or **phase-comparison Monopulse**.
- Phase comparison monopulse uses two antenna beams to obtain an angle measurement in one coordinate. But the two beams cover the same region of space.
- Here the two beams look in the same direction whereas in earlier case they looked at slightly different directions



Amplitude comparison monopulse



Phase comparison monopulse



**Wave front phase relationships in
phase comparison Monopulse
Radar**

- In above **Fig** two antennas are shown separated by a distance **d**.
- The distance to the target is **R** and is assumed large compared with the antenna separation **d**.
- The line of sight to the target makes an angle **θ** to the perpendicular bisector of the line joining the two antennas.
- The distance from antenna 1 to the target is:

$$R_1 = R + (d/2) \sin \theta$$

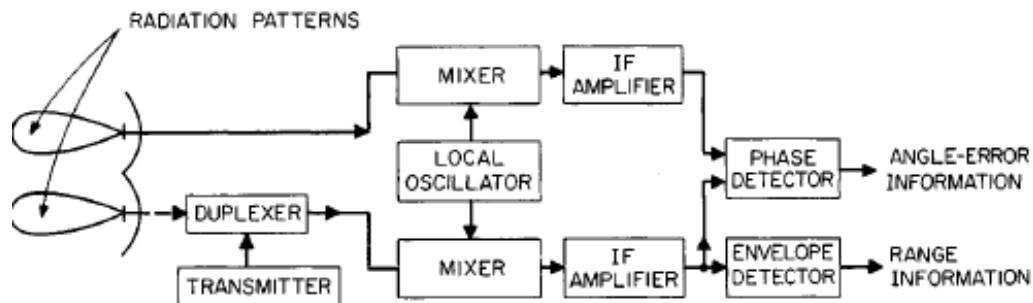
and the distance from antenna 2 to the target is:

$$R_2 = R - (d/2) \sin \theta$$

- The phase difference between the echo signals in the two antennas is approximately:

$$\Delta\theta = (2\pi/\lambda) \cdot d \cdot \sin \theta$$

For small angles where $\sin \theta \approx \theta$, the phase difference is a linear function of the angular error and may be used to position the antenna via a servo-control loop.



Block diagram of a phase comparison monopulse radar (one angle coordinate).

Limitations

- Effect of grating lobes due to wide separation between the antennas
- Ambiguities in the angle measurement
- To overcome these problems, a portion of the parabolic reflectors is sliced off to achieve this.

Important points to remember in amplitude and phase comparison monopulse

- A total of four hybrid junctions generate sum, azimuth and elevation difference channel
- Range information is extracted from the output of the sum channel after amplitude detection
- The angular error signal is obtained by comparing echo amplitudes which actuates a servo mechanism to position the antenna.
- The angle of arrival is determined by comparing the phase difference between signals from two separate antennas
- Antennas of phase comparison are not offset from the axis

Limitations to Tracking Accuracy

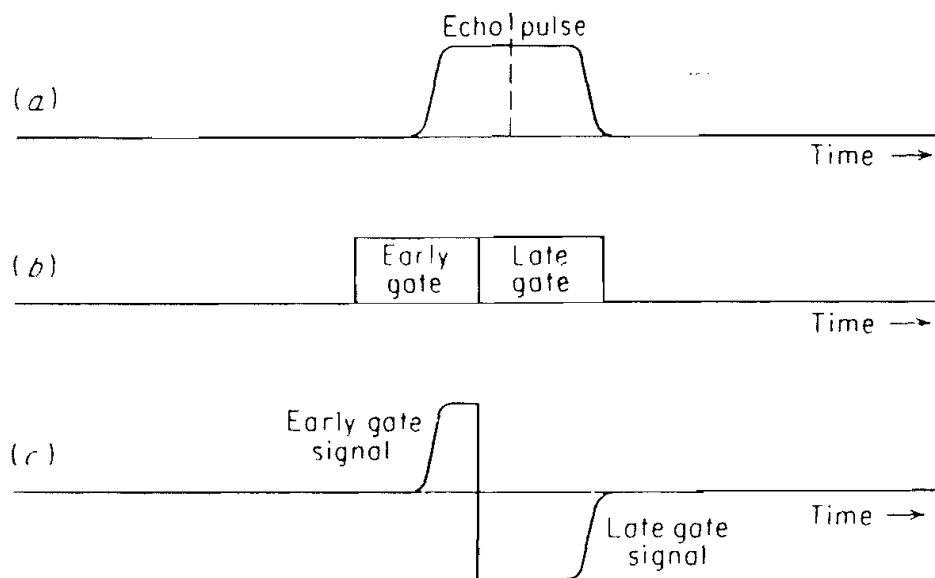
Major effects that determine the accuracy of a tracking radar:

- Glint or angle noise or angular scintillation: which affects all tracking radars especially at short range. The greater the target in angle, the greater is the glint error.
- Receiver noise: affects all radars and mainly determines tracking accuracy at long range.
- RCS or Amplitude fluctuations of the target echo that bother conical scan and sequential lobing trackers but not monopulse.
- Servo noise
- Antenna Beamwidth
- Atmospheric effects

Tracking in range

- In the early days of radar, tracking of target in range was usually done manually by an operator who watched an A-scope or similar presentation and positioned a handwheel to maintain a marker on the display over desired target pip.
- The setting of the handwheel was a measure of the target range and was converted to an electrical signal and supplied to a data processor.
- Manually tracking has many limitations and it cannot be used in systems such as missiles where there is no operator present.
- It was soon replaced by closed loop automatic tracking, such as **split gate tracker**.
- Split gate tracker uses two split range gates called early gate and late gate.
- The echo pulse is shown in Fig.a, the relative position of the gates at a particular instant shown in Fig.b, and the error signal shown in Fig c.
- The portion of the signal energy contained in the early gate is less than that in the late gate.
- If the outputs of the two gates are subtracted, an error signal **shown in fig c** will result which is used to reposition the center of the gates.

Figure : Split-range-gate tracking.(a)Echo pulse (b)early-late range gates(c) difference signal between early and late range gates



- The magnitude of the error signal is a measure of the difference between the center of the pulse and the center of the gates.
- The sign of the error signal determines the direction in which the gates must be repositioned by a feedback-control system.
- When the error signal is zero the range gates are centered on the pulse.
- The range gating necessary to perform automatic tracking in Range offers several advantages as by products.
- It isolates one target, excluding targets at other ranges. This permits the boxcar generator to be employed.
- Also, range gating improves the signal-to-noise ratio since it eliminates the noise from the other range intervals. Hence the width of the gate should be sufficiently narrow to minimize extraneous noise.
- A reasonable compromise is to make the gate width two to five times of the pulse width.

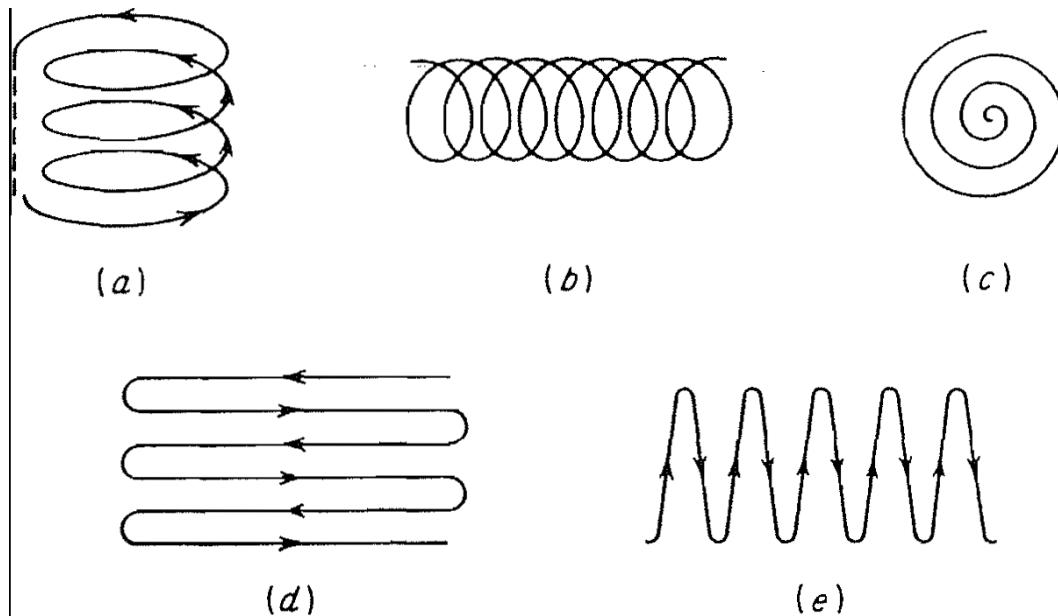
Acquisition and Scanning Patterns

- A tracking radar must first find and acquire its target before it can operate as a tracker.
- Most tracking radars employ a narrow pencil beam for accurate tracking in angle; but it can be difficult to search a large volume targets when using a narrow antenna beamwidth.
- Search must be done with care to cover the entire volume uniformly and efficiently.
- Some other radar, therefore must first find the target to be tracked and then designated the target's coordinates to the tracker. These radars have been called acquisition radars or designation radars that search a large volume.

Types of Scanning Patterns

The purpose of using scanning antenna is to find the direction of the target with respect to the transmitter. The direction of the antenna at the instance when echo is received, gives the direction of location of the target.

Examples of acquisition search patterns: (a) Trace of helical scanning beam (b) Palmer scan (c) spiral scan (d) raster, or TV, scan (e) nodding scan. The raster scan is sometimes called an n -bar scan, where n is the number of horizontal rows.



a) Helical Scanning

- Helical scanning covers a hemisphere.
- In the *helical scan*, the antenna is continuously rotated in azimuth while it is simultaneously raised or lowered in elevation.
- It's typical speed of rotation is 6rpm along with a rise of 20% and was utilized in world war II for anti-aircraft gun batteries as fire controlled radar.

b) Palmer Scan

- The *Palmer scan* consists of a rapid circular scan (conical scan) about the axis of the antenna, combined with a linear movement of the axis of rotation.
- When the axis of rotation is held stationary the Palmer scan reduces to the *conical scan*.
- Because of this feature the palmer scan is used with conical scan tracking radars which must operate in both search and track mode.

c) **Spiral Scan**

- The spiral scan covers an angular search volume with circular symmetry.
- Both the spiral scan and the Palmer scan suffer from the disadvantage that all parts of the scan volume do not receive the same energy unless the scanning speed is varied during the scan cycle.
- As a consequence, the number of hits returned from a target when searching with a constant scanning rate depends upon the position of the target within the search area.

d) **Raster Scan**

- The raster or TV, scan, unlike the Palmer or the spiral scan, scans the search area in a uniform manner.
- The raster scan is a simple and convenient means for searching a limited sector, rectangular in shape.

e) **Nodding Scan**

- The antenna is moved rapidly in elevation while it rotates slowly in azimuth thus scanning in both planes.
- The pattern covers the complete hemisphere i.e. elevation angle extending to 90° and the azimuth scan angle to 360°
- Used in height finding radars

Comparison of Trackers

- Of the four continuous-tracking-radar techniques that have been discussed (sequential lobing, conical scan, amplitude-comparison monopulse, and phase-comparison monopulse), conical scan and amplitude-comparison monopulse have seen more application than the other two.
- In phase comparison four antennas are placed in awkward direction and its side lobe levels are higher than desired.
- Sequential lobing suffers more losses with complex antenna and feed system
- Amplitude comparison has high SNR
- It has higher precision in target tracking due to the absence of target amplitude fluctuations
- Angle error in two coordinates can be obtained by a single pulse
- Conical scan integrates no of pulses and then extracts angle measurement but vice versa in monopulse.

SNR

- The SNR from a monopulse radar is greater than that from a conical scan since it views target at the peak of sum pattern.
- SNR is 2 to 4 db greater.

Accuracy

- Due to high SNR, the range accuracy is also high in monopulse.
- The accuracy is not affected by fluctuations in the amplitude of the echo signal.
- Both systems are degraded by the wandering of the apparent position of the a target caused by glint.

Complexity

- Monopulse is more complex of the two.
- Conical scan has to rotate or nutate the beam at high speed.
- The cassegrain is a popular choice for monopulse
- A space fed phased array can implement monopulse by using a multiple feed similar to cassegrain.

Min No. of Pulses

- A monopulse can perform on the basis of a single pulse. For a phased array one pulse is sufficient
- The conical scan tracker requires a minimum no. of four pulses per revolution of beam to extract an angle measurement in two coordinates.
- The monopulse first makes its angle measurement and then integrates a no. of measurements to obtain the required SNR.
- The conical scan integrates a no. of pulses first and then extracts the angle measurement.

Susceptibility to ECM

- Conical scan tracker is more vulnerable to spoofing that takes advantage of its conical scan frequency
- It can also suffer from deliberate amplitude fluctuations.
- A well designed monopulse is hard to deceive.

Application

- Monopulse trackers should be used when good angle accuracy is needed.
- When high performance tracking is not necessary, the conical scan tracker might be used for its low cost.

Comparison of Monopulse Tracking and Conical Scan Tracking

Monopulse Tracking	Conical Scan Tracking
Multiple beams are used to determine the angle of arrival of the echo signal	A single antenna beam on a time shared basis is used
Single pulse is required to derive angle error information	Multiple pulses are required
High SNR	Low SNR
More accurate tracking	Less accurate tracking
Complex Design	Simple Design
Cassegrain antenna is used	Horn antenna is used
High cost	Low cost

The **Search radar** is usually less precise and only distinguishes between targets that are hundreds of yards or even miles apart. Radar resolution is usually divided into two categories viz. range resolution and angular resolution (i.e. bearing resolution).

- Distance coverage: Long, medium, short ranges (20 km to 2000 km)
- High power density on the target: high peak power, long pulses, long pulse trains, high antenna gain
- Low PRFs
Search options: rapid search rate with narrow beams or slower search rate with wide beams

Tracking Radar

The **Tracking radar** continuously emits the EM waves in the air and detects the targetted object when it comes in the path of the waves.

- * Accurate angle and range measurement required
- * Minimize time on target for rapid processing
- * Special tracking techniques: monopulse, conical scan, beam switching

DETECTION OF RADAR SIGNALS IN NOISE

Introduction

Matched filter receiver

Response characteristics and derivation

Correlation function and cross correlation receiver

Efficiency of Non-matched filters

Matched filter with non-white noise

- The two basic operations performed by radar are (1) **detection** of the presence of reflecting objects, and (2) **extraction** of information from the received waveform to obtain such target data as position, velocity, and perhaps size.
- In this chapter some aspects of the problem of detecting radar signals in the presence of noise will be considered. Noise ultimately limits the capability of any radar.

Matched-Filter Receiver

- A network whose frequency-response function maximizes the output peak-signal-to- mean-noise (power) ratio is called a matched filter. This criterion, or its equivalent, is used for the design of almost all radar receivers.
 - The frequency-response function, denoted $H(f)$, expresses the relative amplitude and phase of the output of a network with respect to the input when the input is a pure sinusoid.
 - The magnitude $|H(f)|$ of the frequency-response function is the receiver amplitude pass band characteristic.
 - If the bandwidth of the receiver pass band is wide compared with that occupied by the signal energy, extraneous noise is introduced by the excess bandwidth which lowers the output signal-to-noise ratio. On the other hand, if the receiver bandwidth is narrower than the bandwidth occupied by the signal, the noise energy is reduced along with a considerable part of the signal energy.
-
- The net result is again a lowered signal-to-noise ratio. Thus there is an optimum bandwidth at which the signal-to-noise ratio is a maximum. This is well known to the radar receiver designer.
 - The rule of thumb quoted in pulse radar practice is that the receiver bandwidth B should be approximately equal to the reciprocal of the pulse width τ . This is a reasonable approximation for pulse radars with conventional superheterodyne receivers. It is not generally valid for other waveforms.
 - The exact specification of the optimum receiver characteristic involves the frequency- response function and the shape of the received waveform.
 - The receiver frequency-response function, is assumed to apply from the antenna terminals to the output of the IF amplifier.
 - The second detector and video portion of the well designed radar superheterodyne receiver will have negligible effect on the output signal-to- noise ratio if the receiver is designed as a matched filter. Narrow banding is most conveniently accomplished in the IF.
 - The bandwidths of the RF and mixer stages of the normal superheterodyne receiver are usually large compared with the IF bandwidth. Therefore the frequency-response function of the portion of the receiver included between the antenna terminals to the output of the IF amplifier is taken to be that of the IF amplifier alone.

- Thus we need only obtain the frequency-response function that maximizes the signal-to-noise ratio at the output of the IF. The IF amplifier may be considered as a filter with gain.
- For a received waveform $s(t)$ with a given ratio of signal energy E to noise energy N_0 (or noise power per hertz of bandwidth), North showed that the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean- noise (power) ratio for a fixed input signal-to-noise (energy) ratio is

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

where $S(f) = \int_{-\infty}^{\infty} s(t) \exp(-j2\pi f t) dt =$ voltage spectrum (Fourier transform) of input signal

$S^*(f) =$ complex conjugate of $S(f)$

$t_1 =$ fixed value of time at which signal is observed to be maximum

$G_a =$ constant equal to maximum filter gain (generally taken to be unity)

- The noise that accompanies the signal is assumed to be stationary and to have a uniform spectrum (white noise). It need not be Gaussian.
- The filter whose frequency-response function is given by Eq. above has been called the North filter, the conjugate filter, or more usually the matched filter. It has also been called the Fourier transform criterion.
- The frequency-response function of the matched filter is the conjugate of the spectrum of the received waveform except for the phase shift $\exp(-j2\pi f t_1)$. This phase shift varies uniformly with frequency. Its effect is to cause a constant time delay.
- The frequency spectrum of the received signal may be written as an amplitude spectrum $|S(f)|$ and a phase spectrum $\exp[-j\phi_s(f)]$.
- The matched- filter frequency-response function may similarly be written in terms of its amplitude and phase spectra $|H(f)|$ and $\exp[-j\phi_m(f)]$. Ignoring the constant G_a , Eq. above for the matched filter may then be written as

$$|H(f)| \exp[-j\phi_m(f)] = |S(f)| \exp\{j[\phi_s(f) - 2\pi f t_1]\}$$

or

$$|H(f)| = |S(f)|$$

and

$$\phi_m(f) = -\phi_s(f) + 2\pi f t_1$$

- Thus the amplitude spectrum of the matched filter is the same as the amplitude spectrum of the signal, but the phase spectrum of the matched filter is the negative of the phase spectrum of the signal plus a phase shift proportional to frequency.

- The matched filter may also be specified by its impulse response $h(t)$, which is the inverse Fourier transform of the frequency-response function.

$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(j2\pi ft) df$$

- Physically, the impulse response is the output of the filter as a function of time when the input is an impulse (delta function).

Since $S^*(f) = S(-f)$, we have

$$h(t) = G_a \int_{-\infty}^{\infty} S(f) \exp[j2\pi f(t_1 - t)] df = G_a s(t_1 - t)$$

- A rather interesting result is that the impulse response of the matched filter is the image of the received waveform; that is, it is the same as the received signal run backward in time starting from the fixed time t_1 .
- Figure 1 shows a received waveform $s(t)$ and the impulse response $h(t)$ of its matched filter. The impulse response of the filter, if it is to be realizable, is not defined for $t < 0$. (One cannot have any response before the impulse is applied.) Therefore we must always have $t < t_1$.
- This is equivalent to the condition placed on the transfer function $H(f)$ that there be a phase shift $\exp(-j2\pi ft_1)$. However, for the sake of convenience, the impulse response of the matched filter is sometimes written simply as $s(-t)$.

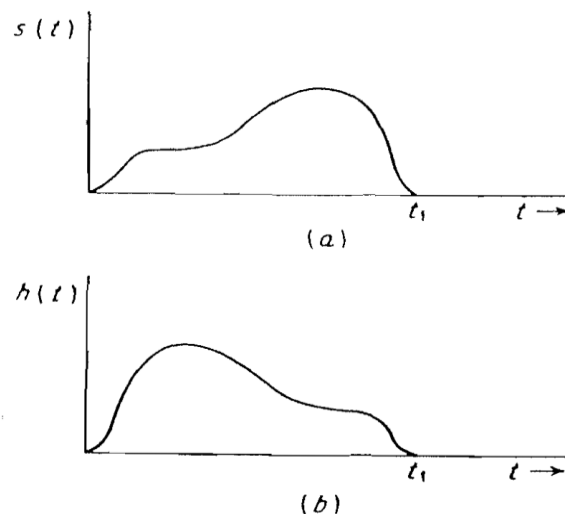


Fig.1 (a) Received waveform $s(t)$; **(b)** impulse response $h(t)$ of the matched filter.

Derivation of the matched-filter characteristic

- The frequency-response function of the matched filter has been derived by a number of authors using either the calculus of variations or the Schwartz inequality. We shall derive the matched-filter frequency-response function using the Schwartz inequality.

- We wish to show that the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean-noise ratio is

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

- When the input noise is stationary and white (uniform spectral density). The ratio we wish to maximize is

$$R_f = \frac{|s_o(t)|_{\max}^2}{N}$$

- Where $|s_o(t)|_{\max}$ = maximum value of output signal voltage and N = mean noise power at receiver output. The ratio R_f is not quite the same as the signal-to-noise ratio which has been considered in the radar equation.

- The output voltage of a filter with frequency-response function $H(f)$ is

$$|s_o(t)| = \left| \int_{-\infty}^{\infty} S(f) H(f) \exp(j2\pi f t) df \right|$$

- Where $S(f)$ is the Fourier transform of the input (received) signal. The mean output noise power is

$$N = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df$$

- Where N_0 is the input noise power per unit bandwidth. The factor appears before the integral because the limits extend from $-\infty$ to $+\infty$, whereas N_0 is defined as the noise power per cycle of bandwidth over positive values only. Assuming that the maximum value of $|s_o(t)|$ occurs at time $t = t_1$, the ratio R_f becomes

$$R_f = \frac{\left| \int_{-\infty}^{\infty} S(f) H(f) \exp(j2\pi f t_1) df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df}$$

Schwartz's inequality states that if P and Q are two complex functions, then

$$\int P^* P dx \int Q^* Q dx \geq \left| \int P^* Q dx \right|^2$$

- The equality sign applies when $P = kQ$, where k is a constant. Letting

$$P^* = S(f) \exp(j2\pi f t_1) \quad \text{and} \quad Q = H(f)$$

and recalling that

$$\int P^* P \, dx = \int |P|^2 \, dx$$

- We get, on applying the Schwartz inequality to the numerator of Eq. earlier, we get

$$R_f \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 \, df \int_{-\infty}^{\infty} |S(f)|^2 \, df}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df} = \frac{\int_{-\infty}^{\infty} |S(f)|^2 \, df}{\frac{N_0}{2}}$$

From Parseval's theorem,

$$\int_{-\infty}^{\infty} |S(f)|^2 \, df = \int_{-\infty}^{\infty} s^2(t) \, dt = \text{signal energy} = E$$

Therefore we have

$$R_f \leq \frac{2E}{N_0}$$

- The frequency-response function which maximizes the peak-signal-to-mean-noise ratio R_f may be obtained by noting that the equality sign in Eq. applies when $P = kQ$, or

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

Where the constant k has been set equal to $1/G_a$.

The matched filter and the correlation function

- The output of the matched filter may be shown to be proportional to the input signal cross-correlated with a replica of the transmitted signal, except for the time delay t_1 .
- The cross correlation function $R(t)$ of two signals $y(\lambda)$ and $s(\lambda)$, each of finite duration, is defined as

$$R(t) = \int_{-\infty}^{\infty} y(\lambda)s(\lambda - t) d\lambda$$

- The output $y_o(t)$ of a filter with impulse response $h(t)$ when the input is $y_{in}(t) = s(t) + n(t)$ is

$$y_o(t) = \int_{-\infty}^{\infty} y_{in}(\lambda)h(t - \lambda) d\lambda$$

- If the filter is a matched filter, then $h(\lambda) = s(t_1 - \lambda)$ and Eq. above becomes

$$y_o(t) = \int_{-\infty}^{\infty} y_{in}(\lambda)s(t_1 - t + \lambda) d\lambda = R(t - t_1)$$

- Thus the matched filter forms the cross correlation between the received signal corrupted by noise and a replica of the transmitted signal.
- The replica of the transmitted signal is "built in" to the matched filter via the frequency-response function.
- If the input signal $y_{in}(t)$ were the same as the signal $s(t)$ for which the matched filter was designed (that is, the noise is assumed negligible), the output would be the autocorrelation function.

Cross correlation receiver(correlation detection)

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda)s(t_1 - t + \lambda) d\lambda = R(t - t_1)$$

- Equation above describes the output of the matched filter as the cross correlation between the input signal and a delayed replica of the transmitted signal.
- This implies that the matched-filter receiver can be replaced by a cross-correlation receiver that performs the same mathematical operation as shown in Fig.

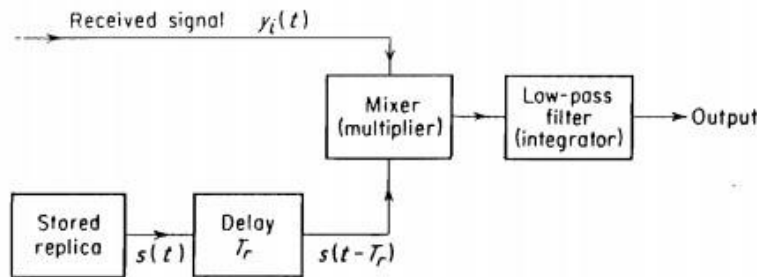
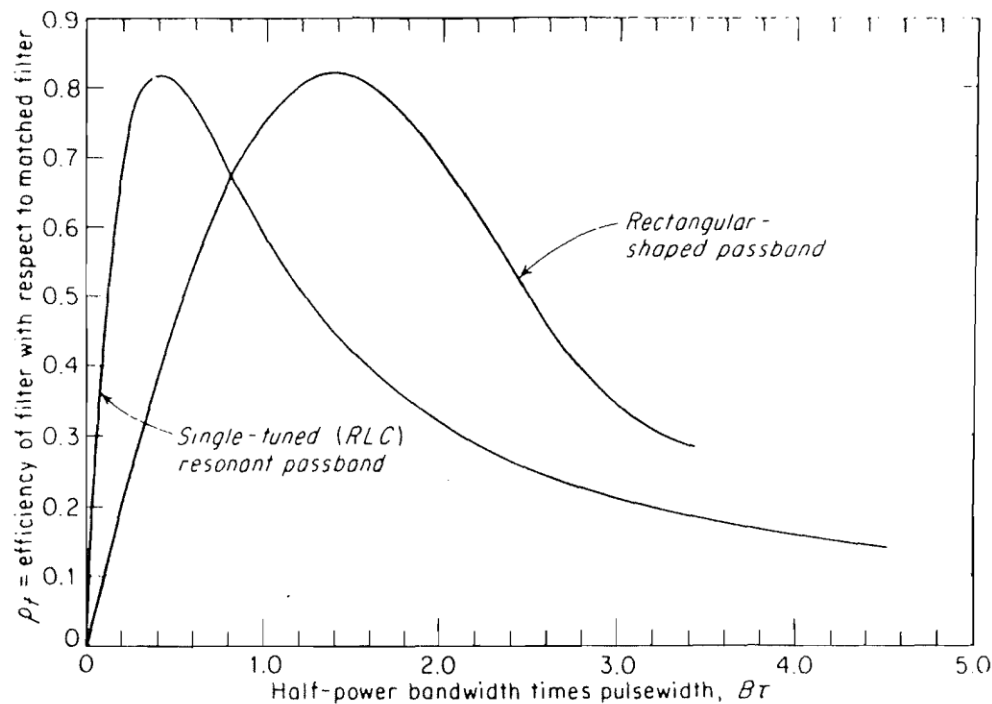


Fig: Block diagram of a cross-correlation receiver

- The input signal $y(t)$ is multiplied by a delayed replica of the transmitted signal $s(t - T_r)$, and the product is passed through a low-pass filter to perform the integration.
- The cross-correlation receiver of above Fig tests for the presence of a target at only a single time delay T_r . Targets at other time delays, or ranges, might be found by varying T_r . However, this requires a longer search time.
- The cross-correlation receiver and the matched-filter receiver are equivalent mathematically, the choice as to which one to use in a particular radar application is determined by which is more practical to implement.
- The matched-filter receiver, or an approximation, has been generally preferred in the vast majority of applications.

Efficiency of non-matched filters

- In practice the matched filter cannot always be obtained exactly. It is appropriate, therefore, to examine the efficiency of non matched filters compared with the ideal matched filter.
- The measure of efficiency is taken as the peak signal-to-noise ratio from the non matched filter divided by the peak signal-to-noise ratio ($2E/N_0$) from the matched filter.
- Figure. Plots the efficiency for a single-tuned (RLC) resonant filter and a rectangular-shaped filter of half-power bandwidth B_τ when the input is a rectangular pulse of width τ
- The maximum efficiency of the single-tuned filter occurs for $B_\tau \approx 0.4$. The corresponding loss in signal-to-noise ratio is 0.88 dB as compared with a matched filter. .



- Table lists the values of $B\tau$ which maximize the signal-to-noise ratio (SNR) for various combinations of filters and pulse shapes. It can be seen that the loss in SNR incurred by use of these non-matched filters is small.

Input signal	Filter	Optimum $B\tau$	Loss in SNR compared with matched filter, dB
Rectangular pulse	Rectangular	1.37	0.85
Rectangular pulse	Gaussian	0.72	0.49
Gaussian pulse	Rectangular	0.72	0.49
Gaussian pulse	Gaussian	0.44	0 (matched)
Rectangular pulse	One-stage, single-tuned circuit	0.4	0.88
Rectangular pulse	2 cascaded single-tuned stages	0.613	0.56
Rectangular pulse	5 cascaded single-tuned stages	0.672	0.5

Table: Efficiency of nonmatched filters compared with the matched filter

Matched filter with nonwhite noise

- In the derivation of the matched-filter characteristic, the spectrum of the noise accompanying the signal was assumed to be white; that is, it was independent of frequency.
- If this assumption were not true, the filter which maximizes the output signal-to-noise ratio would not be the same as the matched filter.
- It has been shown that if the input power spectrum of the interfering noise is given by $[N_i(f)]^2$, the frequency-response function of the filter which maximizes the output signal-to-noise ratio is

$$H(f) = \frac{G_a S^*(f) \exp(-j2\pi f t_1)}{[N_i(f)]^2}$$

- When the noise is nonwhite, the filter which maximizes the output signal-to-noise ratio is called the NWN (nonwhite noise) matched filter.

- For white noise $[N_i(f)]^2 = \text{constant}$ and the NWN matched-filter frequency-response function of Eq. above reduces to that of Eq. discussed earlier in white noise. Equation above can be written as

$$H(f) = \frac{1}{N_i(f)} \times G_a \left(\frac{S(f)}{N_i(f)} \right)^* \exp(-j2\pi f t_1)$$

- This indicates that the NWN matched filter can be considered as the cascade of two filters.
- The first filter, with frequency-response function $1/N_i(f)$, acts to make the noise spectrum uniform, or white. It is sometimes called the whitening filter.
- The second is the matched filter when the input is white noise and a signal whose spectrum is $S(f)/N_i(f)$.

RADAR RECEIVERS

Noise figure and Noise Temperature

Displays-Types

Duplexers-Branch type and Balanced Type

Circulators as duplexers

Phased array Antennas Introduction, Basic concepts, Radiation Pattern

Beam steering and Beam width changes

Series vs parallel feeds

Applications, Advantages and Limitations

Radar Receivers

- The receiver section is connected to the antenna by duplexer. The receiver section is super heterodyne type and it consists of mixer, IF amplifier, detector and video amplifier.
- The first stage of the receiver is usually an RF amplifier with very low noise properties. The RF amplifier is followed by a mixer of fairly low noise figure.
- The mixer is a crystal diode which is fed to IF amplifier. The IF amplifier operates at 30 or 60 MHz or at nearby frequency.
- IF stage provides most of the receiver gain. The IF amplifier should be a low noise device to ensure that the overall noise figure of the receiver does not deteriorate to great extent.
- The down conversion from microwave frequency to IF frequency may be done in number of stages to ensure adequate image frequency suppression.
- The detector is crystal diode whose output is amplified by video amplifier having the same bandwidth as the IF amplifier.
- The output signal is then fed to a display unit which may be a CRT.

Characteristics of radar receiver

- The function of the radar receiver is to detect desired echo signals in the presence of noise, interference, or clutter.
- It must separate wanted from unwanted signals, and amplify the wanted signals to a level where target information can be displayed to an operator or used in an automatic data processor.
- The design of the radar receiver will depend not only on the type of waveform to be detected, but on the nature of the noise, interference, and clutter echoes with which the desired echo signals must compete.
- Noise can enter the receiver via the antenna terminals along with the desired signals, or it might be generated within the receiver itself. At the microwave frequencies usually used for radar, the external noise which enters via the antenna is generally quite low. The measure of receiver internal noise is the noise-figure.
- Good receiver design is based on maximizing the output signal-to-noise ratio. To maximize the output signal-to-noise ratio, the receiver must be designed as a matched filter, or its equivalent. The matched filter specifies the frequency response function of the IF part of the radar receiver.
- Obviously, the receiver should be designed to generate as little internal noise as possible, especially in the input stages where the desired signals are the weakest.

- Receiver design also must be concerned with achieving sufficient gain, phase, and amplitude stability, dynamic range, tuning, ruggedness, and simplicity.
- Protection must be provided against overload or saturation, and burnout from nearby interfering transmitters. Timing and reference signals are needed to properly extract target information.
- Timing and reference signals are needed to properly extract target information. Specific applications such as MTI radar, tracking radar, or radars designed to minimize clutter place special demands on the receiver.
- Receivers that must operate with a transmitter whose frequency can drift need some means of automatic frequency control (AFC).
- Radars that encounter hostile counter-measures need receivers that can minimize the effects of such interference.
- Thus there can be many demands placed upon the receiver designer in meeting the requirements of modern high-quality radar systems.
- Although the super regenerative, crystal video and tuned radio frequency (TRF) receivers have been employed in radar systems, the superheterodyne has seen almost exclusive application because of its good sensitivity, high gain, selectivity, and reliability. No other receiver type has been competitive to the superheterodyne.

Receiver Noise Figure

Noise figure:

Noise figure of a receiver is a measure of the noise produced by a practical receiver as compared with the noise of an ideal receiver. The noise figure F_n may be defined as:

$$F_n = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} = \frac{N_{out}}{kT_0 B_n G} \quad \text{Eq.(1)}$$

Where S_{in} = available input signal power
 N_{in} = available input noise power (equal to $kT_0 B$)
 S_{out} = available output signal power
 N_{out} = available output noise power

"Available power" refers to the power which would be delivered to a matched load. The available gain G is equal to S_{out}/S_{in} .

k = Boltzmann's constant = 1.38×10^{-23} J/deg, T_0 = standard temperature of 290 K (approximately room temperature) and B_n is the noise bandwidth (defined earlier).

The product $kT_0 \approx 4 \times 10^{-21}$ W/Hz. The purpose of defining a standard temperature is to refer any measurements to a common basis of comparison.

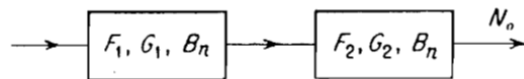
- Equation above permits two different but equivalent interpretations of noise figure.
- It may be considered as the degradation of the signal-to-noise ratio caused by the network (receiver), or it may be interpreted as the ratio of the actual available output noise power to the noise power which would be available if the network merely amplified the thermal noise.
- The noise figure may also be written

$$F_n = \frac{kT_0 B_n G + \Delta N}{kT_0 B_n G} = 1 + \frac{\Delta N}{kT_0 B_n G}$$

- where ΔN is the additional noise introduced by the network itself. The noise figure is commonly expressed in decibels, that is, $10 \log F_n$. The term noise factor is also used at times instead of noise figure. The two terms are synonymous.

Noise figure of networks in cascade

- Consider two networks in cascade, each with the same noise bandwidth B_n but with different noise figures and available gain (Fig).
- Let F_1, G_1 be the noise figure and available gain, respectively, of the first network, and F_2, G_2 be similar parameters for the second network.
- The problem is to find F_o , the overall noise-figure of the two circuits in cascade.



- From the definition of noise figure the output noise N_o of the two circuits in cascade is

$N_o = F_o G_1 G_2 k T_0 B_n$ = Noise from network 1 at output of network2 + Noise ΔN_2 introduced by network2

$$N_o = k T_0 B_n F_1 G_1 G_2 + \Delta N_2 = k T_0 B_n F_1 G_1 G_2 + (F_2 - 1) k T_0 B_n G_2$$

$$F_o = F_1 + \frac{F_2 - 1}{G_1}$$

- The contribution of the second network to the overall noise-figure may be made negligible if the gain of the first network is large. This is of importance in the design of multistage receivers.
- It is not sufficient that only the first stage of a low-noise receiver have a small noise figure.
- The succeeding stage must also have a small noise figure, or else the gain of the first stage must be high enough to swamp the noise of the succeeding stage if the first network is not an amplifier but is a network with loss (as in a crystal mixer), the gain GI should be interpreted as a number less than unity.
- Extending the same method, the noise figure of N networks in cascade may be shown to be:

$$F_o = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{G_1 G_2 \cdots G_{N-1}}$$

Noise temperature

- The noise introduced by a network may also be expressed as an **effective Noise temperature**, T_e defined as that (fictional) temperature at the input of the network which would account for the noise ΔN at the output. Therefore $\Delta N = kT_e B_0 G$ and

$$F_n = 1 + \frac{T_e}{T_0}$$

$$T_e = (F_n - 1)T_0$$

- The system noise temperature T_s is defined as the effective noise temperature of the receiver system including the effects of antenna temperature T_a . (It is also sometimes called the system operating noise temperature) If the receiver effective noise temperature is T_e , then

$$T_s = T_a + T_e = T_0 F_s$$

- where F_s is the system noise-figure including the effect of antenna temperature. The effective noise temperature of a receiver consisting of a number of networks in cascade is

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots$$

where T_i and G_i , are the effective noise temperature and gain of the i th network.

- The effective noise temperature and the noise figure both describe the same characteristic of a network. In general, the effective noise temperature has been preferred for describing low-noise devices, and the noise figure is preferred for conventional receivers.

Displays-Types

- The purpose of the display is to visually present the information contained in the radar echo signal in a form suitable for operator interpretation and action.
- When the display is connected directly to the video output of the receiver, the information displayed is called raw video. This is the "traditional" type of radar presentation.
- When the receiver video output is first processed by an automatic detector or automatic detection and tracking processor (ADT), the output displayed is sometimes called synthetic video.
- The cathode-ray tube (CRT) has been almost universally used as the radar display. There are two basic cathode-ray tube displays. One is the deflection-modulated CRT, such as the A-scope, in which a target is indicated by the deflection of the electron beam.
- The other is the intensity modulated CRT, such as the PPI, in which a target is indicated by intensifying the electron beam and presenting a luminous spot on the face of the CRT.

- In general, deflection-modulated displays have the advantage of simpler circuits than those of intensity-modulated displays, and targets may be more readily discerned in the presence of noise or interference.
- On the other hand, intensity-modulated displays have the advantage of presenting data in a convenient and easily interpreted form.
- The deflection of the beam or the appearance of an intensity-modulated spot on a radar display caused by the presence of a target is commonly referred to as a blip.

Types of display presentations: The various types of CRT displays which might be used for surveillance and tracking radars are defined as follows:

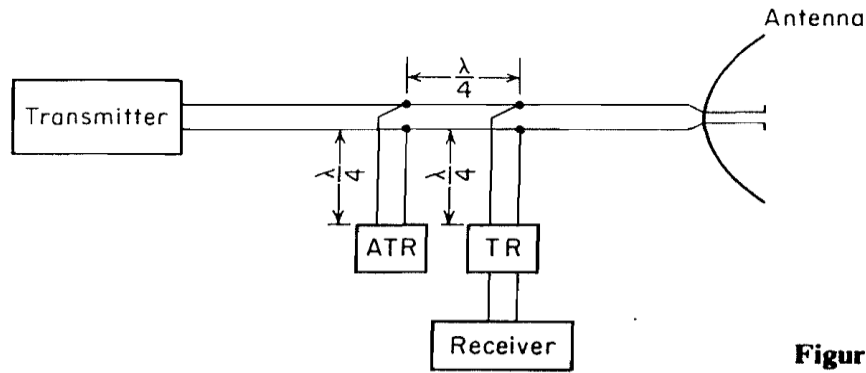
- **A-scope:** A deflection-modulated display in which the vertical deflection is proportional to target echo strength and the horizontal coordinate is proportional to range.
- **B-scope:** An intensity-modulated rectangular display with azimuth angle indicated by the horizontal coordinate and range by the vertical coordinate.
- **C-scope:** An intensity-modulated rectangular display with azimuth angle indicated by the horizontal coordinate and elevation angle by the vertical coordinate.
- **D-scope:** A C-scope in which the blips extend vertically to give a rough estimate of distance.
- **E-scope:** An intensity-modulated rectangular display with distance indicated by the horizontal coordinate and elevation angle by the vertical coordinate. Similar to the RHI in which target height or altitude is the vertical coordinate.
- **F-Scope:** A rectangular display in which a target appears as a centralized blip when the radar antenna is aimed at it. Horizontal and vertical aiming errors are respectively indicated by the horizontal and vertical displacement of the blip.
- **G-Scope:** A rectangular display in which a target appears as a laterally centralized blip when the radar antenna is aimed at it in azimuth, and wings appear to grow on the pip as the distance to the target is diminished; horizontal and vertical aiming errors are respectively indicated by horizontal and vertical displacement of the blip.
- **H-scope:** A B-scope modified to include indication of angle of elevation. The target appears as two closely spaced blips which approximate a short bright line, the slope of which is in proportion to the sine of the angle of target elevation.

- **I-scope:** A display in which a target appears as a complete circle when the radar antenna is pointed at it and in which the radius of the circle is proportional to target distance; incorrect aiming of the antenna changes the circle to a segment whose arc length is inversely proportional to the magnitude of the pointing error, and the position of the segment indicates the reciprocal of the pointing direction of the antenna.
 - **J-scope:** A modified A-scope in which the time base is a circle and targets appear as radial deflections from the time base.
 - **K-scope:** A modified A-scope in which a target appears as a pair of vertical deflections. When the radar antenna is correctly pointed at the target, the two deflections are of equal height, and when not so pointed, the difference in deflection amplitude is an indication of the direction and magnitude of the pointing error.
 - **L-scope:** A display in which a target appears as two horizontal blips, one extending to the right from a central vertical time base and the other to the left; both blips are of equal amplitude when the radar is pointed directly at the target, any inequality representing relative pointing error, and distance upward along the baseline representing target distance.
-
- **M-scope:** A type of A-scope in which the target distance is determined by moving an adjustable pedestal signal along the baseline until it coincides with the horizontal position of the target signal deflections; the control which moves the pedestal is calibrated in distance.
 - **N-scope:** A K-scope having an adjustable pedestal signal, as in the M-scope, for the measurement of distance.
 - **O-scope:** An A-scope modified by the inclusion of an adjustable notch for measuring distance.
 - **PPI or Plan Position Indicator (also called P-scope):** An intensity-modulated circular display on which echo signals produced from reflecting objects are shown in plan position with range and azimuth angle displayed in polar (rho-theta) coordinates, forming a map-like display. An offset, or off center, PPI has the zero position of the time base at a position other than at the center of the display to provide the equivalent of a larger display for a selected portion of the service area. A delayed PPI is one in which the initiation of the time base is delayed.
 - **R-scope:** An A-scope with a segment of the time base expanded near the blip for greater accuracy in distance measurement.
 - **RHI or Range-Height Indicator:** An intensity modulated display with height (altitude) as the vertical axis and range as the horizontal axis.
 - **PPI, A-scope, B-scope, and RHI** are among the more usual displays employed in radar.

Duplexers

The duplexer is the device that 1) switches the radar antenna to either transmitter or the receiver 2) serves to protect the receiver from burnout or damage during the transmission.

Branch type Duplexer

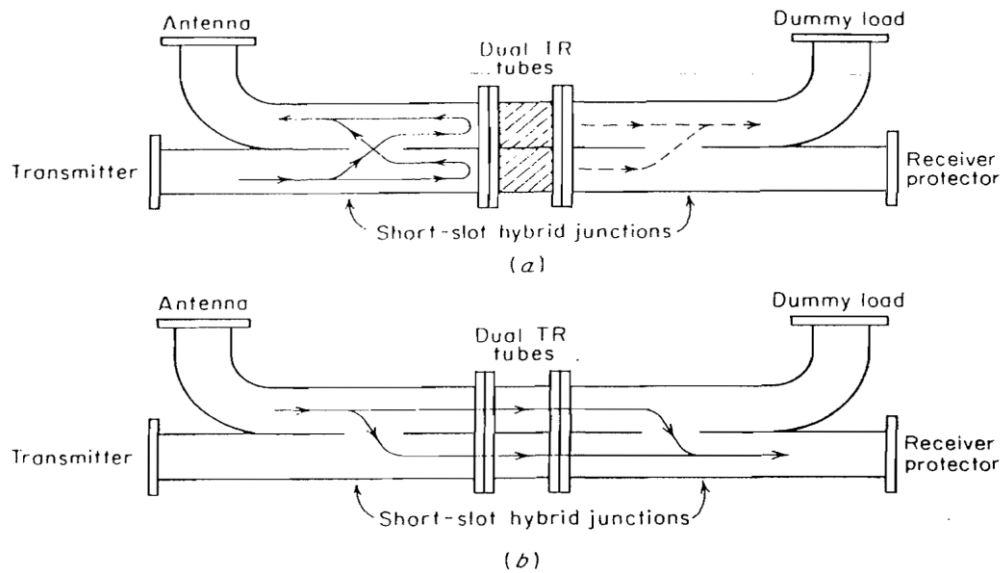


Figure

Figure : Principle of branch-type duplexer

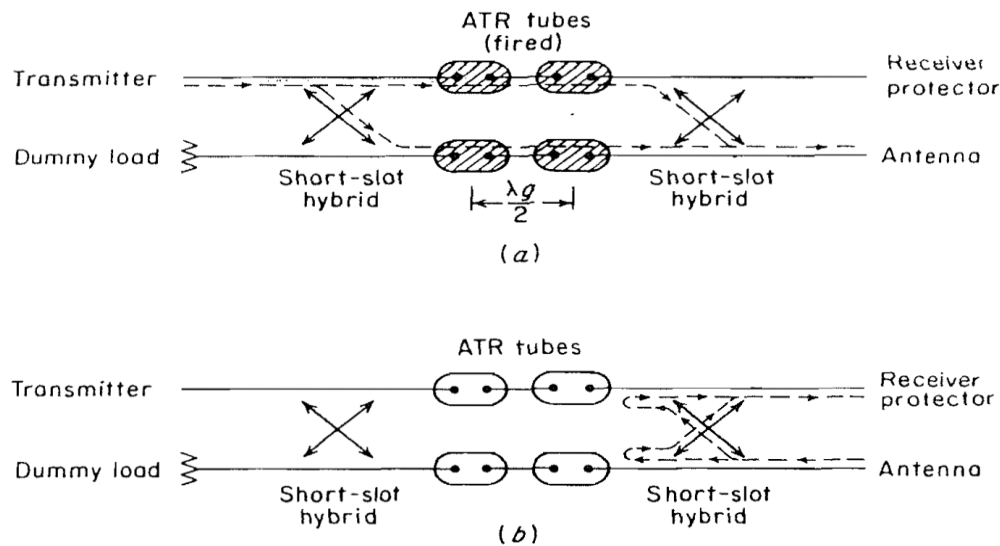
- The branch-type duplexer, shown in above Fig is one of the earliest duplexer configurations. It consists of a TR (transmit-receive) switch and an ATR (anti-transmit receive) switch, both of which are gas-discharge tubes.
- When the transmitter is turned on, the TR and the ATR tubes ionize; that is, they break down, or fire. The TR in the fired condition acts as a short circuit to prevent transmitter power from entering the receiver.
- Since the TR is located a quarter wavelength from the main transmission line, it appears as a short circuit at the receiver but as an open circuit at the transmission line so that it does not impede the flow of transmitter power.
- Since the ATR is displaced a quarter wavelength from the main transmission line, the short circuit it produces during the fired condition appears as an open circuit on the transmission line and thus has no effect on transmission.
- During reception, the transmitter is off and neither the TR nor the ATR is fired.
- The open circuit of the ATR, being a quarter wave from the transmission line, appears as a short circuit across the line.
- Since this short circuit is located a quarter wave from the receiver branch line, the transmitter is effectively disconnected from the line and the echo signal power is directed to the receiver.
- The branch-type duplexer is of limited bandwidth and power handling capability, and has generally been replaced by the balanced duplexer and other protection devices. It is used, inspite of these limitations, in some low-cost radars.

- **Balanced duplexer**



**Figure : Balanced duplexer using dual TR tubes and two short-slot hybrid junctions.
(a) Transmit Condition (b) Receive condition.**

- The balanced duplexer is based on the short-slot hybrid junction which consists of two sections of waveguides joined along one of their narrow walls with a slot cut in the common narrow wall to provide coupling between the two.
- The short-slot hybrid may be considered as a broadband directional coupler with a coupling ratio of 3 dB.
- In the transmit condition (Figure a) power is divided equally into each waveguide by the first short slot hybrid junction. Both TR tubes break down and reflect the incident power out the antenna arm as shown.
- *The short-slot hybrid has the property that each time the energy passes through the slot in either direction, its phase is advanced 90° . Therefore, the energy travels as indicated by the solid lines.*
- Any energy which leaks through the TR tubes (shown by the dashed lines) is directed to the arm with the matched dummy load and not to the receiver. In addition to the attenuation provided by the TR tubes, the hybrid junctions provide an additional 20 to 30 dB of isolation.
- On reception the TR tubes are unfired and the echo signals pass through the duplexer and into the receiver as shown in Fig.6b. The power splits equally at the first junction and because of the 90° phase advance on passing through the slot, the energy recombines in the receiving arm and not in the dummy-load arm.
- The power-handling capability of the balanced duplexer is inherently greater than that of the branch-type duplexer and it has wide bandwidth, over ten percent with proper design. A *receiver protector*, is usually inserted between the duplexer and the receiver for added protection.



Balanced duplexer using ATR tubes.
(a) Transmit condition (b) receive condition

CIRCULATORS AS DUPLEXERS

CIRCULATOR AND RECEIVER PROTECTOR

- The ferrite circulator is a three or four-port device that can in principle, offer separation of the transmitter and receiver without the need for the conventional duplexer configurations explained earlier.
- The circulator does not provide sufficient protection by itself and requires a receiver protector like duplexers.
- The isolation between the transmitter and receiver ports of a circulator is seldom sufficient to protect the receiver from damage.
- However, it is not the isolation between transmitter and receiver ports that usually determines the amount of transmitter power at the receiver, but the impedance mismatch at the antenna which reflects transmitter power back into the receiver.
- The VSWR is a measure of the amount of power reflected by the antenna. For example, a VSWR of 1.5 means that about 4 percent of the transmitter power will be reflected by the antenna mismatch in the direction of the receiver, which corresponds to an isolation of only 14 dB. About 11 percent of the power is reflected when the VSWR is 2.0, corresponding to less than 10 dB of isolation. Thus, a receiver protector is almost always required.

- It also reduces to safe level radiations from nearby transmitters.
- The receiver protector might use solid-state diodes for an all solid-state configuration, or it might be a passive TR-limiter consisting of a radioactive primed TR-tube followed by a diode limiter.
- The ferrite circulator with receiver protector is attractive for radar applications because of its long life, wide bandwidth, and compact design.

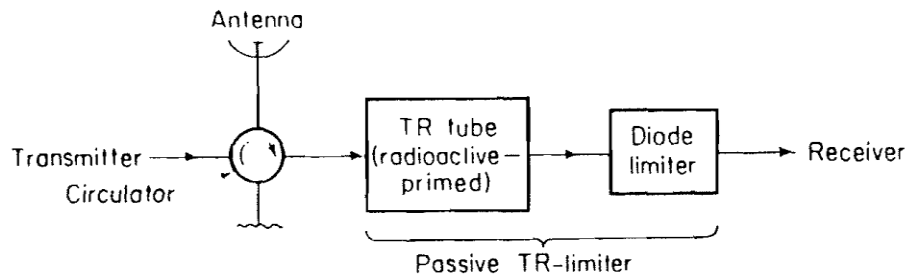


Figure : circulator and receiver protector. A four-port circulator is shown with the fourth port terminated in a matched load to provide greater isolation between the transmitter and the receiver than provided by a three-port circulator

Introduction to phased array antennas

- The phased array is a directive antenna made up of individual radiating antennas, or elements, which generate a radiation pattern whose shape and direction is determined by the relative phases and amplitudes of the currents at the individual elements.
- By properly varying the relative phases it is possible to steer the direction of the radiation.
- The radiating elements might be dipoles open-ended waveguides, slots cut in waveguide, or any other type of antenna.
- Became of interest to Radar due to the inherent flexibility it has offered in steering the beam by means of electronic control rather than by physical movement of the antenna.
- It has been considered in those radar applications where it is necessary to shift the beam rapidly from one position in space to another, or where it is required to obtain information about many targets at a flexible, rapid data rate.
- The full potential of a phased-array antenna requires the use of a computer that can determine in real time, on the basis of the actual operational situation, how best to use the capabilities offered by the array.

- Initially In World War 2, the United States, Great Britain, and Germany used radar with fixed phased-array antennas in which the beam was scanned by mechanically actuated phase shifters.
- A major advance in phased array technology was made in the early 1950s with the replacement of mechanically actuated phase shifters by electronic phase shifters.
- Frequency scanning in one angular coordinate was the first successful electronic scanning technique to be applied.
- The introduction of digitally switched phase shifters employing either ferrites or diodes in the early 1960s made a significant improvement in the practicality of phased arrays that could be electronically steered in two orthogonal angular coordinates.

Basic concept

- An array antenna consists of a number of individual radiating elements suitably spaced with respect to one another.
- Two common geometrical forms of array antennas used in radar are the linear array and the planar array.

Linear array

- A *linear array* consists of elements arranged in a straight line in one dimension.
- The linear array generates a fan beam when the phase relationships are such that the radiation is perpendicular to the array.
- When the radiation is at some angle other than broadside, the radiation pattern is a conical-shaped beam.
- The linear array can also act as a feed for a parabolic-cylinder antenna.

Planar array

- A *planar array* is a two dimensional configuration of elements arranged to lie in a plane. The planar array may be thought of as a linear array of linear arrays.
- The two-dimensional planar array is the most commonly used in radar applications since it is fundamentally the most versatile of all radar antennas.
- A rectangular aperture can produce a fan shaped beam. A square or a circular aperture produces a pencil beam.
- The array can be made to simultaneously generate many search and/or tracking beams with the same aperture.

Broadside array

- A *broadside array* is one in which the direction of maximum radiation is perpendicular, or almost perpendicular to the line (or plane) of the array.
- The broadside linear-array antenna may be used where broad coverage in one plane and narrow beam width in the orthogonal plane are desired.

End fire

- An *end fire* array has its maximum radiation parallel to the array.
- The end fire array is a special case of the linear or the planar array when the beam is directed along the array.
- End fire linear arrays have not been widely used in radar applications.
- They are usually limited to low or medium gains since an end fire linear antenna of high gain requires an excessively long array.
- Small end fire arrays are sometimes used as the radiating elements of a broadside array if directive elements are required.

Other types of array antennas

- An array whose elements are distributed on a non planar surface is called a *conformal array*.
- An array in which the relative phase shift between elements is controlled by electronic devices is called an *electronically scanned array*.
- In an electronically scanned array the antenna elements, the transmitters, the receivers, and the data-processing portions of the radar are often designed as a unit.

Radiation pattern

- Consider a linear array made up of N elements equally spaced a distance d apart as shown in **Fig.**

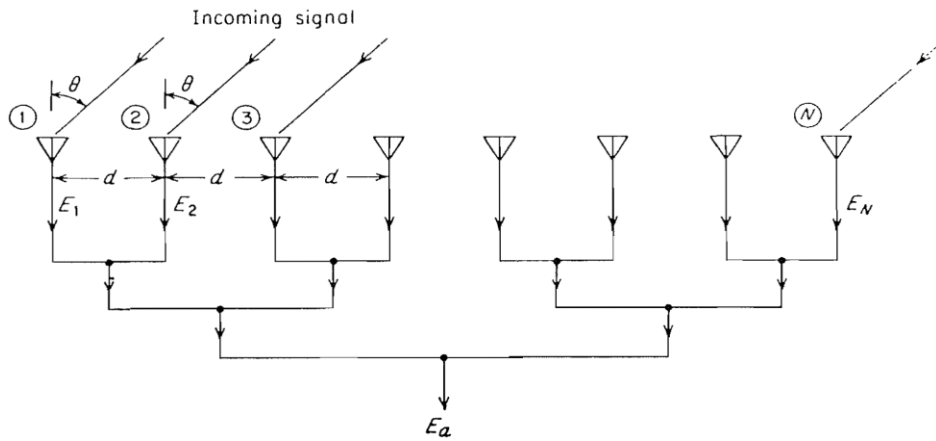


Figure : N-element linear array.

- The elements are assumed to be isotropic point sources radiating uniformly in all directions with equal amplitude and phase.
- The outputs of all the elements are summed via lines of equal length to give a sum output voltage E_a .
- Element 1 will be taken as the reference signal with zero phase. The difference in the phase of the signals in adjacent elements is $\Psi = 2\pi (d/\lambda) \sin \theta$, where θ is the direction of the incoming radiation.
- It is further assumed that the amplitudes and phases of the signals at each element are weighted uniformly. Therefore the amplitudes of the voltages in each element are the same and, for convenience, will be taken to be unity.
- The sum of all the voltages from the individual elements, when the phase difference between adjacent elements is Ψ , can be written as

$$E_a = \sin \omega t + \sin (\omega t + \psi) + \sin (\omega t + 2\psi) + \cdots + \sin [\omega t + (N - 1)\psi]$$

where ω is the angular frequency of the signal. The sum can be written as

$$E_a = \sin \left[\omega t + (N - 1) \frac{\psi}{2} \right] \frac{\sin (N\psi/2)}{\sin (\psi/2)}$$

- The first factor is a sine wave of frequency ω with a phase shift $(N - 1) \psi/2$. The second term represents the amplitude factor of the form $\sin (N\psi/2)/\sin (\psi/2)$. The field intensity pattern is the magnitude of the equation 2 , or

$$|E_a(\theta)| = \left| \frac{\sin [N\pi(d/\lambda) \sin \theta]}{\sin [\pi(d/\lambda) \sin \theta]} \right|$$

- The pattern has nulls whenever the numerator is zero.
- $N\pi(d/\lambda)\sin\theta = 0, \pm \pi, \pm 2\pi, \dots, \pm n\pi$, where $n = \text{integer}$. The denominator, however, is zero when $\pi(d/\lambda) \sin\theta = 0, \pm \pi, \pm 2\pi, \dots, \pm n\pi$. Note that when the denominator is zero, the numerator is also zero.
- The value of the field intensity pattern is indeterminate when both the denominator and numerator are zero. However, by applying L'Hopital's rule (differentiating numerator and denominator separately) it is found that $|E_a|$ is a maximum whenever $\sin\theta = \pm n\lambda/d$.
- These maxima all have the same value and are equal to N . The maximum at $\sin\theta = 0$ defines the main beam. The other maxima are called grating lobes. They are generally undesirable and are to be avoided.

- The radiation pattern is equal to the normalized square of the amplitude, or

$$G_a(\theta) = \frac{|E_a|^2}{N^2} = \frac{\sin^2 [N\pi(d/\lambda) \sin \theta]}{N^2 \sin^2 [\pi(d/\lambda) \sin \theta]}$$

- When directive elements are used, the resultant array antenna radiation pattern is

$$G(\theta) = G_e(\theta) \frac{\sin^2 [N\pi(d/\lambda) \sin \theta]}{N^2 \sin^2 [\pi(d/\lambda) \sin \theta]} = G_e(\theta) G_a(\theta)$$

- where $G_e(\theta)$ is the radiation pattern of an individual element. The resultant radiation pattern is the product of the element factor $G_e(\theta)$ and the array factor $G_a(\theta)$.
- Grating lobes caused by a widely spaced array may therefore be eliminated with directive elements which radiate little or no energy in the directions of the undesired lobes.
- For example, when the element spacing $d = 2\lambda$, grating lobes occur at $\theta = \pm 30^\circ$ and $\pm 90^\circ$ in addition to the main beam at $\theta = 0^\circ$. If the individual elements have a beamwidth somewhat less than 60° , the grating lobes of the array factor will be suppressed.

- In a two-dimensional, rectangular planar array, the radiation pattern may sometimes be written as the product of the radiation patterns in the two planes which contain the principal axes of the antenna.
- If the radiation patterns in the two principal planes are $G_1(\theta_e)$ and $G_2(\theta_a)$ the two-dimensional antenna pattern is

$$G(\theta_e, \theta_a) = G_1(\theta_e)G_2(\theta_a)$$

- Thus, the normalized radiation pattern of a uniformly illuminated rectangular array is

$$G(\theta_e, \theta_a) = \frac{\sin^2 [N\pi(d/\lambda) \sin \theta_a] \sin^2 [M\pi(d/\lambda) \sin \theta_e]}{N^2 \sin^2 [\pi(d/\lambda) \sin \theta_a] M^2 \sin^2 [\pi(d/\lambda) \sin \theta_e]}$$

- Where N = number of radiating elements in θ_a dimension with spacing d and M the number in θ_e dimension.

Beam steering phased array antennas

- The beam of an array antenna may be steered rapidly in space without moving large mechanical masses by properly varying the phase of the signals applied to each element.
- Consider an array of equally spaced elements. The spacing between adjacent elements is d , and the signals at each element are assumed to be of equal amplitude.
- If the same phase is applied to all elements, the relative phase difference between adjacent elements is zero and the position of the main beam will be broadside to the array at an angle $\theta = 0$.
- The main beam will point in a direction other than broadside if the relative phase difference between elements is other than zero.
- The direction of the main beam is at an angle θ_0 , when the phase difference is $\theta = 2\pi (d/\lambda) \sin \theta_0$. The phase at each element is therefore $(\theta_c + m \theta)$ where $m = 0, 1, 2, \dots (N - 1)$ and θ_c is any constant phase applied to all elements.

- The normalized radiation pattern of the array when the phase difference between adjacent elements is θ is given by:

$$G(\theta) = \frac{\sin^2 [N\pi(d/\lambda)(\sin \theta - \sin \theta_0)]}{N^2 \sin^2 [\pi(d/\lambda)(\sin \theta - \sin \theta_0)]}$$

- The maximum of the radiation pattern occurs when $\sin \theta = \sin \theta_0$
- The above Equation states that the main beam of the antenna pattern may be positioned to an angle θ_0 by the insertion of the proper phase shift θ at each element of the array. If variable, rather than fixed, phase shifters are used, the beam may be steered as the relative phase between elements is changed

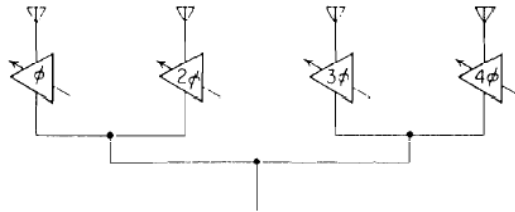


FIG: Steering of an antenna beam with variable phase shifters (parallel-fed array).

- Using an argument similar to the non scanning array described previously, grating lobes appear at an angle θ_g whenever the denominator is zero, or when

$$\text{or } \pi \frac{d}{\lambda} (\sin \theta_g - \sin \theta_0) = \pm n\pi$$

$$|\sin \theta_g - \sin \theta_0| = \pm n \frac{\lambda}{d}$$

- If a grating lobe is permitted to appear at -90° when the main beam is steered to $+90^\circ$, it is found from the above that $d = \lambda/2$.
- Thus the element spacing must not be larger than a half wavelength if the beam is to be steered over a wide angle without having undesirable **grating** lobes appear.
- Practical array antennas do not scan $\pm 90^\circ$. If the scan is limited to $\pm 60^\circ$ the element spacing should be less than 0.54λ .

Change of beam width with steering angle

- The half-power beam width in the plane of scan increases as the beam is scanned off the broadside direction. The beam width is approximately inversely proportional to $\cos \theta_0$, where θ_0 is the angle measured from the normal to the antenna.
- This may be proved by assuming that the sine in the denominator of $G(\theta)$ discussed earlier can be replaced by its argument, so that the radiation pattern is of the form $(\sin^2 u)/u^2$, where $u = N\pi (d/\lambda)(\sin \theta - \sin \theta_0)$.
- The $(\sin^2 u)/u^2$ antenna pattern is reduced to half its maximum value when $u = \pm 0.443\pi$. Denote by θ_+ the angle corresponding to the half-power point when $\theta > \theta_0$, and θ_- , the angle corresponding to the half-power point when $\theta < \theta_0$; that is, θ_+ corresponds to $u = +0.443\pi$ and θ_- to $u = -0.443\pi$.
- The $\sin \theta - \sin \theta_0$, term in the expression for u can be written

$$\sin \theta - \sin \theta_0 = \sin (\theta - \theta_0) \cos \theta_0 - [1 - \cos (\theta - \theta_0)] \sin \theta_0$$

- The second term on the right-hand side of Eq. above can be neglected when θ_0 is small (beam is near broadside), so that

$$\sin \theta - \sin \theta_0 \approx \sin (\theta - \theta_0) \cos \theta_0$$

- Using the above approximation, the two angles corresponding to the 3-dB points of the antenna pattern are

$$\theta_+ - \theta_0 = \sin^{-1} \left(\frac{0.443\lambda}{Nd \cos \theta_0} \right) \approx \frac{0.443\lambda}{Nd \cos \theta_0}$$

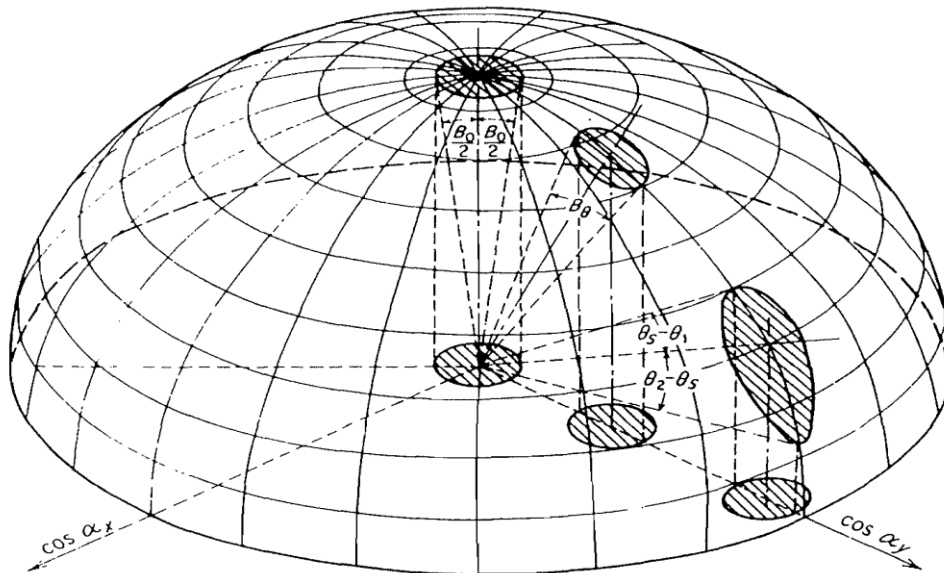
$$\theta_- - \theta_0 = \sin^{-1} \left(\frac{-0.443\lambda}{Nd \cos \theta_0} \right) \approx \frac{-0.443\lambda}{Nd \cos \theta_0}$$

- The half-power beamwidth is

$$\theta_B = \theta_+ - \theta_- \approx \frac{0.886\lambda}{Nd \cos \theta_0}$$

- Therefore, when the beam is positioned an angle θ_0 off broadside, the beamwidth in the plane of scan increases as $(\cos \theta_0)^{-1}$.

- The variation of the beam shape with scan angle is graphically shown in **Fig below**.
- The antenna radiation pattern is plotted in spherical coordinates as a function of the two direction cosines, $\cos \alpha_x$ and $\cos \alpha_y$ of the radius vector specifying the point of observation.
- The angle θ is measured from the $\cos \alpha_x$ axis, and θ is measured from the axis perpendicular to the $\cos \alpha_x$ and $\cos \alpha_y$ axes.
- In **Fig.** θ is taken to be a constant value of 90° and the beam is scanned in the θ coordinate.
- At $\theta = 0$ (beam broad side to the array) a symmetrical pencil beam of half-power width B_0 is assumed.
- The shape of the beam at the other angular positions is the projection of the circular beam shape on the surface of the unit sphere.
- It can be seen that as the beam is scanned in the θ direction, it broadens in that direction, but is constant in the θ direction.
- For $\theta \neq 0$, the beam shape is not symmetrical about the center of the beam, but is eccentric.



Beam width and eccentricity of the scanned beam

Applications of phased array antennas

The phased array antenna has seen application in radar for a wide variety of purposes:

- Aircraft surveillance from on board ship
- Satellite surveillance
- Ballistic missile defense
- Air defense
- Aircraft landing systems
- Mortar and artillery location
- Tracking of ballistic missiles and Airborne bomber radar (EAR).
- Many developmental array radars have been developed and built in USA. Although much effort and funds have been spent on this activity, ***except for limited-scan arrays*** there has not been any large serial production of such radars compared to the serial production of radars with mechanically rotating reflector antennas.

Advantages phased array antennas.

- **Inertialess rapid beam-steering:** The beam from an array can be scanned, or switched from one position to another, in a time limited only by the switching speed of the phase shifters. Typically, the beam can be switched in several microseconds, but it can be considerably shorter if desired.
- **Multiple, independent beams:** A single aperture can generate many simultaneous independent beams. Alternatively, the same effect can be obtained by rapidly switching a single beam through a sequence of positions.
- **Potential for large peak and / or average power:** If necessary, each element of the array can be fed by a separate high-power transmitter with the combining of the outputs made in space to obtain a total power greater than can be obtained from a single transmitter.

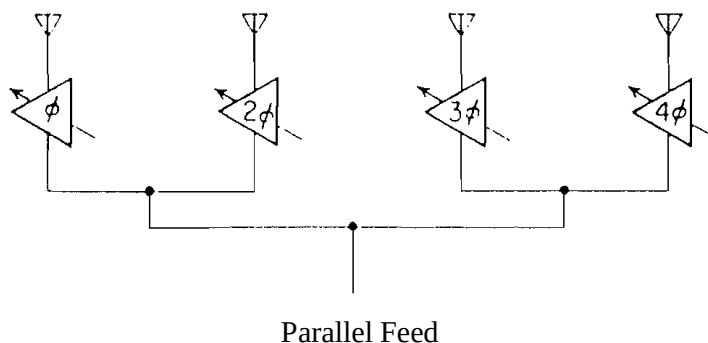
- **Control of the radiation pattern.** A particular radiation pattern may be more readily obtained with the array than with other microwave antennas since the amplitude and phase of each array element may be individually controlled. Thus, radiation patterns with extremely low side lobes or with a shaped main beam may be achieved conveniently. Separate monopulse sum and difference patterns, each with its own optimum shape, can also be generated.
- **Graceful degradation.** The distributed nature of the array means that it can fail only gradually and not all at once (catastrophically).
- **Convenient aperture shape.** The shape of the array permits flush mounting and it can be strengthened to resist blast.
- **Electronic beam stabilization.** The ability to steer the beam electronically can be used to stabilize the beam direction when the radar is on an unstable platform, such as a ship or aircraft that is subject to roll, pitch, and yaw disturbances.

Limitations of phased array antennas

1. The major limitation that has limited the widespread use of the conventional phased array in radar is its high cost, which is due in large part to its complexity.
2. When graceful degradation has gone too far a separate maintenance is needed.
3. When a planar array is electronically scanned, the change of mutual coupling that accompanies a change in beam position makes the maintenance of low sidelobes more difficult.
4. Although the array has the potential for radiating large power, it is seldom that an array is required to radiate more power than can be radiated by other antenna types or to utilize a total power which cannot possibly be generated by current high-power microwave tube technology that feeds a single transmission line.

Series Vs Parallel feeds

- The phase relationship between the adjacent elements of the array can be obtained with either series fed or parallel fed arrangement.
 - In series fed arrangement, the energy may be transmitted from one end to the line or it may be fed from the center out to each end.
 - The adjacent elements are connected by a phase shifter with phase ϕ .
 - All the phase shifters are identical and introduce the same amount of phase shift which is less than 2π radians.
 - In parallel fed the energy to be radiated is divided between the elements by a power splitter.
 - When a series of power splitters are used to create a tree like structure is called a corporate feed.
 - Equal lengths of line transmit the energy to each element so that no unwanted phase difference are introduced by the lines themselves.
 - The maximum phase change required of each phase shifter in the parallel-fed array is many times 2π **radians**.
-
- In a series-fed array containing N phase shifters, the signal suffers the insertion loss of a single phase shifter N times. In a parallel-fed array the insertion loss of the phase shifter is introduced effectively but once.
 - Since each phase shifter in the series-fed linear array of below Fig has **the same value of** phase shift, only a single control signal is needed to steer the beam. The N-element parallel-fed linear array similar to that of below Fig requires a separate control signal for each phase shifter or N - 1 **total**.



Series arrangements for applying phase relationships in an array.
(a) fed from one end; (b) *center-fed*.

